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PyWavelets is open source wavelet transform software for Python. It combines a simple high level interface with low level C and Cython performance.

PyWavelets is very easy to use and get started with. Just install the package, open the Python interactive shell and type:

```python
>>> import pywt
>>> cA, cD = pywt.dwt([1, 2, 3, 4], 'db1')
```

Voila! Computing wavelet transforms has never been so simple :)

Here is a slightly more involved example of applying a digital wavelet transform to an image:

```python
import numpy as np
import matplotlib.pyplot as plt
import pywt
import pywt.data

# Load image
original = pywt.data.camera()

# Wavelet transform of image, and plot approximation and details
titles = ['Approximation', ' Horizontal detail', ' Vertical detail', ' Diagonal detail']
coeffs2 = pywt.dwt2(original, 'bior1.3')
LL, (LH, HL, HH) = coeffs2
fig = plt.figure(figsize=(12, 3))
for i, a in enumerate([LL, LH, HL, HH]):
    ax = fig.add_subplot(1, 4, i + 1)
    ax.imshow(a, interpolation="nearest", cmap=plt.cm.gray)
    ax.set_title(titles[i], fontsize=10)
    ax.set_xticks([])
    ax.set_yticks([])
fig.tight_layout()
plt.show()
```
The main features of PyWavelets are:

- 1D, 2D and nD Forward and Inverse Discrete Wavelet Transform (DWT and IDWT)
- 1D, 2D and nD Multilevel DWT and IDWT
- 1D, 2D and nD Stationary Wavelet Transform (Undecimated Wavelet Transform)
- 1D and 2D Wavelet Packet decomposition and reconstruction
- 1D Continuous Wavelet Transform
- Computing Approximations of wavelet and scaling functions
- Over 100 built-in wavelet filters and support for custom wavelets
- Single and double precision calculations
- Real and complex calculations
- Results compatible with Matlab Wavelet Toolbox (TM)
CHAPTER 2

Getting help

Use GitHub Issues, StackOverflow, or the PyWavelets discussions group to post your comments or questions.
PyWavelets is a free Open Source software released under the MIT license.
If you use PyWavelets in a scientific publication, we would appreciate citations of the project via the following JOSS publication:


Specific releases can also be cited via Zenodo. The DOI below will correspond to the most recent release. DOIs for past versions can be found by following the link in the badge below to Zenodo:
5.1 Installing

The latest release, including binary packages for Windows, macOS and Linux, is available for download from PyPI. You can also find source releases at the Releases Page.

You can install PyWavelets with:

```
pip install PyWavelets
```

Users of the Anaconda Python distribution may wish to obtain pre-built Windows, Intel Linux or macOS / OSX binaries from the main or conda-forge channel:

```
conda install pywavelets
```

Several Linux distributions have their own packages for PyWavelets, but these tend to be moderately out of date. Query your Linux package manager tool for `python-pywavelets`, `python-wavelets`, `python-pywt` or a similar package name.

5.1.1 Building from source

The most recent development version can be found on GitHub at https://github.com/PyWavelets/pywt.

The latest release, is available for download from PyPI or on the Releases Page.

If you want or need to install from source, you will need a working C compiler (any common one will work) and a recent version of Cython. Navigate to the PyWavelets source code directory (containing `setup.py`) and type:

```
pip install .
```

The requirements needed to build from source are:

- Python 2.7 or >=3.4
- NumPy >= 1.13.3
To run all the tests for PyWavelets, you will also need to install the Matplotlib package. If SciPy is available, FFT-based continuous wavelet transforms will use the FFT implementation from SciPy instead of NumPy.

See also:

*Development guide* section contains more information on building and installing from source code.

## 5.2 API Reference

### 5.2.1 Wavelets

**Wavelet families**

```python
pywt.families(short=True)
```

Returns a list of available built-in wavelet families.

Currently the built-in families are:

- Haar (`haar`)
- Daubechies (`db`)
- Symlets (`sym`)
- Coiflets (`coif`)
- Biorthogonal (`bior`)
- Reverse biorthogonal (`rbio`)
- “Discrete” FIR approximation of Meyer wavelet (`dmey`)
- Gaussian wavelets (`gaus`)
- Mexican hat wavelet (`mexh`)
- Morlet wavelet (`morl`)
- Complex Gaussian wavelets (`cgau`)
- Shannon wavelets (`shan`)
- Frequency B-Spline wavelets (`fbsp`)
- Complex Morlet wavelets (`cmor`)

**Parameters**


**Returns**

- `families` [list] List of available wavelet families.

**Examples**
Built-in wavelets - \texttt{wavelist()}

\texttt{pywt.wavelist(family=None, kind='all')}

Returns list of available wavelet names for the given family name.

**Parameters**

- \texttt{family} [str, optional] Short family name. If the family name is None (default) then names of all the built-in wavelets are returned. Otherwise the function returns names of wavelets that belong to the given family. Valid names are:

  - 'haar', 'db', 'sym', 'coif', 'bior', 'rbio', 'dmey', 'gaus', 'mexh', 'morl', 'cgau', 'shan', 'fbsp', 'cmor'

- \texttt{kind} [{'all', 'continuous', 'discrete'}, optional] Whether to return only wavelet names of discrete or continuous wavelets, or all wavelets. Default is 'all'. Ignored if \texttt{family} is specified.

**Returns**

- \texttt{wavelist} [list of str] List of available wavelet names.

**Examples**

```python
>>> import pywt
>>> pywt.wavelist('coif')
['coif1', 'coif2', 'coif3', 'coif4', 'coif5', 'coif6', 'coif7', ...
>>> pywt.wavelist(kind='continuous')
['cgau1', 'cgau2', 'cgau3', 'cgau4', 'cgau5', 'cgau6', 'cgau7', ...
```

Custom discrete wavelets are also supported through the \texttt{Wavelet} object constructor as described below.

**Wavelet object**

\texttt{class pywt.Wavelet(name[, filter_bank=None])}

Describes properties of a discrete wavelet identified by the specified wavelet name. For continuous wavelets see \texttt{pywt.ContinuousWavelet} instead. In order to use a built-in wavelet the \texttt{name} parameter must be a valid wavelet name from the \texttt{pywt.wavelist()} list.

Custom Wavelet objects can be created by passing a user-defined filters set with the \texttt{filter_bank} parameter.

**Parameters**

- \texttt{name} – Wavelet name
- \texttt{filter_bank} – Use a user supplied filter bank instead of a built-in \texttt{Wavelet}. 

5.2. API Reference
The filter bank object can be a list of four filters coefficients or an object with `filter_bank` attribute, which returns a list of such filters in the following order:

```
[dec_lo, dec_hi, rec_lo, rec_hi]
```

Wavelet objects can also be used as a base filter banks. See section on *using custom wavelets* for more information.

**Example:**

```
>>> import pywt
>>> wavelet = pywt.Wavelet('db1')
```

**name**

Wavelet name.

**short_name**

Short wavelet name.

**dec_lo**

Decomposition filter values.

**dec_hi**

Decomposition filter values.

**rec_lo**

Reconstruction filter values.

**rec_hi**

Reconstruction filter values.

**dec_len**

Decomposition filter length.

**rec_len**

Reconstruction filter length.

**filter_bank**

Returns filters list for the current wavelet in the following order:

```
[dec_lo, dec_hi, rec_lo, rec_hi]
```

**inverse_filter_bank**

Returns list of reverse wavelet filters coefficients. The mapping from the `filter_coeffs` list is as follows:

```
[rec_lo[::-1], rec_hi[::-1], dec_lo[::-1], dec_hi[::-1]]
```

**short_family_name**

Wavelet short family name.

**family_name**

Wavelet family name.

**orthogonal**

Set if wavelet is orthogonal.

**biorthogonal**

Set if wavelet is biorthogonal.

**symmetry**

asymmetric, near symmetric, symmetric
vanishing_moments_psi
Number of vanishing moments for the wavelet function

vanishing_moments_phi
Number of vanishing moments for the scaling function

Example:

```python
>>> def format_array(arr):
...     return ','.join(['%.14f' % x for x in arr])

>>> import pywt
>>> wavelet = pywt.Wavelet('db1')
>>> print(wavelet)
Wavelet db1
Family name: Daubechies
Short name: db
Filters length: 2
Orthogonal: True
Biorthogonal: True
Symmetry: asymmetric
DWT: True
CWT: False

>>> print(format_array(wavelet.dec_lo), format_array(wavelet.dec_hi))
[0.70710678118655, 0.70710678118655] [-0.70710678118655, 0.70710678118655]

>>> print(format_array(wavelet.rec_lo), format_array(wavelet.rec_hi))
[0.70710678118655, 0.70710678118655] [0.70710678118655, -0.70710678118655]
```

### Approximating wavelet and scaling functions - Wavelet.wavefun()

Wavelet.wavefun(level)

Changed in version 0.2: The time (space) localisation of approximation function points was added.

The `wavefun()` method can be used to calculate approximations of scaling function (\(\phi\)) and wavelet function (\(\psi\)) at the given level of refinement.

For **orthogonal** wavelets returns approximations of scaling function and wavelet function with corresponding x-grid coordinates:

\[
[\phi, \psi, x] = \text{wavelet.wavefun}(\text{level})
\]

Example:

```python
>>> import pywt
>>> wavelet = pywt.Wavelet('db2')
>>> phi, psi, x = wavelet.wavefun(level=5)
```

For other (**biorthogonal** but not **orthogonal**) wavelets returns approximations of scaling and wavelet function both for decomposition and reconstruction and corresponding x-grid coordinates:

\[
[\phi_d, \psi_d, \phi_r, \psi_r, x] = \text{wavelet.wavefun}(\text{level})
\]

Example:

```python
>>> import pywt
>>> wavelet = pywt.Wavelet('bior3.5')
>>> phi_d, psi_d, phi_r, psi_r, x = wavelet.wavefun(level=5)
```
Using custom wavelets

PyWavelets comes with a long list of the most popular wavelets built-in and ready to use. If you need to use a specific wavelet which is not included in the list it is very easy to do so. Just pass a list of four filters or an object with a filter_bank attribute as a filter_bank argument to the Wavelet constructor.

The filters list, either in a form of a simple Python list or returned via the filter_bank attribute, must be in the following order:

- lowpass decomposition filter
- highpass decomposition filter
- lowpass reconstruction filter
- highpass reconstruction filter

just as for the filter_bank attribute of the Wavelet class.

The Wavelet object created in this way is a standard Wavelet instance.

The following example illustrates the way of creating custom Wavelet objects from plain Python lists of filter coefficients and a filter_bank-like object.

Example:

```python
>>> import pywt, math
>>> c = math.sqrt(2)/2
>>> dec_lo, dec_hi, rec_lo, rec_hi = [c, c], [-c, c], [c, c], [c, -c]
>>> filter_bank = [dec_lo, dec_hi, rec_lo, rec_hi]
>>> myWavelet = pywt.Wavelet(name="myHaarWavelet", filter_bank=filter_bank)
>>> class HaarFilterBank(object):
    ...
    @property
    ...
    def filter_bank(self):
    ...
    c = math.sqrt(2)/2
    ...
    dec_lo, dec_hi, rec_lo, rec_hi = [c, c], [-c, c], [c, c], [c, -c]
    ...
    return [dec_lo, dec_hi, rec_lo, rec_hi]
>>> filter_bank = HaarFilterBank()
>>> myOtherWavelet = pywt.Wavelet(name="myHaarWavelet", filter_bank=filter_bank)
```

ContinuousWavelet object

**class** pywt.ContinuousWavelet (name, dtype=np.float64)

Describes properties of a continuous wavelet identified by the specified wavelet name. In order to use a built-in wavelet the name parameter must be a valid wavelet name from the pywt.wavelist() list.

**Parameters**

- **name** – Wavelet name
- **dtype** – numpy.dtype to use for the wavelet. Can be numpy.float64 or numpy.float32.
Example:

```python
>>> import pywt
>>> wavelet = pywt.ContinuousWavelet('gaus1')
```

<table>
<thead>
<tr>
<th>name</th>
<th>Continuous Wavelet name.</th>
</tr>
</thead>
<tbody>
<tr>
<td>short_family_name</td>
<td>Wavelet short family name</td>
</tr>
<tr>
<td>family_name</td>
<td>Wavelet family name</td>
</tr>
<tr>
<td>orthogonal</td>
<td>Set if wavelet is orthogonal</td>
</tr>
<tr>
<td>biorthogonal</td>
<td>Set if wavelet is biorthogonal</td>
</tr>
<tr>
<td>complex_cwt</td>
<td>Returns if wavelet is complex</td>
</tr>
<tr>
<td>lower_bound</td>
<td>Set the lower bound of the effective support</td>
</tr>
<tr>
<td>upper_bound</td>
<td>Set the upper bound of the effective support</td>
</tr>
<tr>
<td>center_frequency</td>
<td>Set the center frequency for the shan, fbsp and cmor wavelets</td>
</tr>
<tr>
<td>bandwidth_frequency</td>
<td>Set the bandwidth frequency for the shan, fbsp and cmor wavelets</td>
</tr>
<tr>
<td>fbsp_order</td>
<td>Set the order for the fbsp wavelet</td>
</tr>
<tr>
<td>symmetry</td>
<td>asymmetric, near symmetric, symmetric, anti-symmetric</td>
</tr>
</tbody>
</table>

Example:

```python
>>> import pywt
>>> wavelet = pywt.ContinuousWavelet('gaus1')
>>> print(wavelet)
ContinuousWavelet gaus1
    Family name: Gaussian
    Short name: gaus
    Symmetry: anti-symmetric
    DWT: False
    CWT: True
    Complex CWT: False
```

Approximating wavelet functions - `ContinuousWavelet.wavefun()`

`ContinuousWavelet.wavefun(level, length = None)`

The `wavefun()` method can be used to calculate approximations of scaling function \(\psi\) with grid \(x\). The vector length is set by `length`. The vector length can also be defined by \(2^\ast level\) if `length` is not set.
For `complex_cwt` wavelets returns a complex approximations of wavelet function with corresponding x-grid coordinates:

```
[psi, x] = wavelet.wavefun(level)
```

Example:

```python
>>> import pywt

>>> wavelet = pywt.ContinuousWavelet('gaus1')

>>> psi, x = wavelet.wavefun(level=5)
```

# Approximating wavelet functions - `ContinuousWavelet.wavefun()`

`pywt.DiscreteContinuousWavelet` *(name[, filter_bank = None]*)

The `DiscreteContinuousWavelet()` returns a Wavelet or a ContinuousWavelet object depending on the given name.

Example:

```python
>>> import pywt

>>> wavelet = pywt.DiscreteContinuousWavelet('db1')

>>> print(wavelet)
Wavelet db1
Family name: Daubechies
Short name: db
Filters length: 2
Orthogonal: True
Biorthogonal: True
Symmetry: asymmetric
DWT: True
CWT: False

>>> wavelet = pywt.DiscreteContinuousWavelet('gaus1')

>>> print(wavelet)
ContinuousWavelet gaus1
Family name: Gaussian
Short name: gaus
Symmetry: anti-symmetric
DWT: False
CWT: True
Complex CWT: False
```

## 5.2.2 Signal extension modes

Because the most common and practical way of representing digital signals in computer science is with finite arrays of values, some extrapolation of the input data has to be performed in order to extend the signal before computing the Discrete Wavelet Transform using the cascading filter banks algorithm.

Depending on the extrapolation method, significant artifacts at the signal’s borders can be introduced during that process, which in turn may lead to inaccurate computations of the `DWT` at the signal’s ends.

PyWavelets provides several methods of signal extrapolation that can be used to minimize this negative effect:

- **zero** - `zero-padding` - signal is extended by adding zero samples:
... 0 0 | x1 x2 ... xn | 0 0 ...

- **constant** - **constant-padding** - border values are replicated:

... x1 x1 | x1 x2 ... xn | xn xn ...

- **symmetric** - **symmetric-padding** - signal is extended by mirroring samples. This mode is also known as half-sample symmetric:

... x2 x1 | x1 x2 ... xn | xn xn-1 ...

- **reflect** - **reflect-padding** - signal is extended by reflecting samples. This mode is also known as whole-sample symmetric:

... x3 x2 | x1 x2 ... xn | xn-1 xn-2 ...

- **periodic** - **periodic-padding** - signal is treated as a periodic one:

... xn-1 xn | x1 x2 ... xn | x1 x2 ...

- **smooth** - **smooth-padding** - signal is extended according to the first derivatives calculated on the edges (straight line)

- **antisymmetric** - **anti-symmetric padding** - signal is extended by mirroring and negating samples. This mode is also known as half-sample anti-symmetric:

... -x2 -x1 | x1 x2 ... xn | -xn -xn-1 ...

- **antireflect** - **anti-symmetric-reflect padding** - signal is extended by reflecting anti-symmetrically about the edge samples. This mode is also known as whole-sample anti-symmetric:

... (2*x1 - x3) (2*x1 - x2) | x1 x2 ... xn | (2*xn - xn-1) (2*xn - xn-2) ...

**DWT** performed for these extension modes is slightly redundant, but ensures perfect reconstruction. To receive the smallest possible number of coefficients, computations can be performed with the **periodization** mode:

- **periodization** - **periodization** - is like **periodic-padding** but gives the smallest possible number of decomposition coefficients. **IDWT** must be performed with the same mode.

**Example:**

```python
>>> import pywt
>>> print(pywt.Modes.modes)
['zero', 'constant', 'symmetric', 'periodic', 'smooth', 'periodization',
 'reflect', 'antisymmetric', 'antireflect']
```

The following figure illustrates how a short signal (red) gets extended (black) outside of its original extent. Note that periodization first extends the signal to an even length prior to using periodic boundary conditions.

"""A visual illustration of the various signal extension modes supported in PyWavelets. For efficiency, in the C routines the array is not actually extended as is done here. This is just a demo for easier visual explanation of the behavior of the various boundary modes.

In practice, which signal extension mode is beneficial will depend on the signal characteristics. For this particular signal, some modes such as..."""

(continues on next page)
"periodic", "antisymmetric" and "zero" result in large discontinuities that would lead to large amplitude boundary coefficients in the detail coefficients of a discrete wavelet transform.

```python
import numpy as np
from matplotlib import pyplot as plt
from pywt._doc_utils import boundary_mode_subplot

# synthetic test signal
x = 5 - np.linspace(-1.9, 1.1, 9)**2

# Create a figure with one subplots per boundary mode
fig, axes = plt.subplots(3, 3, figsize=(10, 6))
plt.subplots_adjust(hspace=0.5)
axes = axes.ravel()
boundary_mode_subplot(x, 'symmetric', axes[0], symw=False)
boundary_mode_subplot(x, 'reflect', axes[1], symw=True)
boundary_mode_subplot(x, 'periodic', axes[2], symw=False)
boundary_mode_subplot(x, 'antisymmetric', axes[3], symw=False)
boundary_mode_subplot(x, 'antireflect', axes[4], symw=True)
boundary_mode_subplot(x, 'periodization', axes[5], symw=False)
boundary_mode_subplot(x, 'smooth', axes[6], symw=False)
boundary_mode_subplot(x, 'constant', axes[7], symw=False)
boundary_mode_subplot(x, 'zero', axes[8], symw=False)
plt.show()
```

Notice that you can use any of the following ways of passing wavelet and mode parameters:

```python
>>> import pywt
>>> (a, d) = pywt.dwt([1, 2, 3, 4, 5, 6], 'db2', 'smooth')
```
>>> (a, d) = pywt.dwt([1,2,3,4,5,6], pywt.Wavelet('db2'), pywt.Modes.smooth)

Note: Extending data in context of PyWavelets does not mean reallocation of the data in the computer’s physical memory and copying values, but rather computing the extra values only when they are needed. This feature saves extra memory and CPU resources and helps to avoid page swapping when handling relatively big data arrays on computers with low physical memory.

Naming Conventions

The correspondence between PyWavelets edge modes and the extension modes available in Matlab’s dwtmode and numpy’s pad are tabulated here for reference.

<table>
<thead>
<tr>
<th>PyWavelets</th>
<th>Matlab</th>
<th>numpy.pad</th>
</tr>
</thead>
<tbody>
<tr>
<td>symmetric</td>
<td>sym, symh</td>
<td>symmetric</td>
</tr>
<tr>
<td>reflect</td>
<td>symw</td>
<td>reflect</td>
</tr>
<tr>
<td>smooth</td>
<td>spd, sp1</td>
<td>N/A</td>
</tr>
<tr>
<td>constant</td>
<td>sp0</td>
<td>edge</td>
</tr>
<tr>
<td>zero</td>
<td>zpd</td>
<td>constant, cval=0</td>
</tr>
<tr>
<td>periodic</td>
<td>ppd</td>
<td>wrap</td>
</tr>
<tr>
<td>periodization</td>
<td>per</td>
<td>N/A</td>
</tr>
<tr>
<td>antisymmetric</td>
<td>asym, asymh</td>
<td>N/A</td>
</tr>
<tr>
<td>antireflect</td>
<td>asymw</td>
<td>reflect, reflect_type='odd'</td>
</tr>
</tbody>
</table>

Padding using PyWavelets Signal Extension Modes - pad

pywt.pad(x, pad_widths, mode)

Extend a 1D signal using a given boundary mode.

This function operates like numpy.pad() but supports all signal extension modes that can be used by PyWavelets discrete wavelet transforms.

Parameters

- x [ndarray] The array to pad
- pad_widths [[sequence, array_like, int]] Number of values padded to the edges of each axis.
  - ((before_1, after_1), ... (before_N, after_N)) unique pad widths for each axis. ((before, after),) yields same before and after pad for each axis.
  - (pad,) or int is a shortcut for before = after = pad width for all axes.
- mode [str, optional] Signal extension mode, see Modes.

Returns

- pad [ndarray] Padded array of rank equal to array with shape increased according to pad_widths.

Notes

The performance of padding in dimensions > 1 may be substantially slower for modes 'smooth' and 'antisymmetric' as these modes are not supported efficiently by the underlying numpy.pad() func-
tion.

Note that the behavior of the 'constant' mode here follows the PyWavelets convention which is different from NumPy (it is equivalent to mode='edge' in numpy.pad()).

PyWavelets provides a function, \texttt{pad()}, that operate like \texttt{numpy.pad()}, but supporting the PyWavelets signal extension modes discussed above. For efficiency, the DWT routines in PyWavelets do not explicitly create padded signals using this function. It can be used to manually prepad signals to reduce boundary effects in functions such as \texttt{cwt()} and \texttt{swt()} that do not currently support all of these signal extension modes.

### 5.2.3 Discrete Wavelet Transform (DWT)

Wavelet transform has recently become a very popular when it comes to analysis, de-noising and compression of signals and images. This section describes functions used to perform single- and multilevel Discrete Wavelet Transforms.

#### Single level dwt

\[ \texttt{pywt.dwt(data, wavelet, mode='symmetric', axis=-1)} \]

Single level Discrete Wavelet Transform.

**Parameters**

- \texttt{data} [array_like] Input signal
- \texttt{wavelet} [Wavelet object or name] Wavelet to use
- \texttt{mode} [str, optional] Signal extension mode, see \texttt{Modes}.
- \texttt{axis} : int, optional Axis over which to compute the DWT. If not given, the last axis is used.

**Returns**

- \texttt{(cA, cD)} [tuple] Approximation and detail coefficients.

**Notes**

Length of coefficients arrays depends on the selected mode. For all modes except periodization:

\[
\text{len(cA)} = \text{len(cD)} = \text{floor}((\text{len(data)} + \text{wavelet.dec_len} - 1) / 2)
\]

For periodization mode ("per"):

\[
\text{len(cA)} = \text{len(cD)} = \text{ceil}((\text{len(data)} / 2)
\]

#### Examples

```python
>>> import pywt
>>> (cA, cD) = pywt.dwt([1, 2, 3, 4, 5, 6], 'db1')
>>> cA
array([ 2.12132034, 4.94974747, 7.77817459])
>>> cD
array([-0.70710678, -0.70710678, -0.70710678])
```

See the \texttt{signal extension modes} section for the list of available options and the \texttt{dwt_coeff_len()} function for information on getting the expected result length.

The transform can be performed over one axis of multi-dimensional data. By default this is the last axis. For multi-dimensional transforms see the \texttt{2D transforms} section.
**Multilevel decomposition using wavedec**

```python
pywt.wavedec(data, wavelet, mode='symmetric', level=None, axis=-1)
```
Multilevel 1D Discrete Wavelet Transform of data.

**Parameters**

- `data`: array_like, Input data
- `wavelet`: [Wavelet object or name string], Wavelet to use
- `mode`: [str, optional], Signal extension mode, see *Modes*.
- `level`: [int, optional], Decomposition level (must be >= 0). If level is None (default) then it will be calculated using the `dwt_max_level` function.
- `axis`: int, optional, Axis over which to compute the DWT. If not given, the last axis is used.

**Returns**

- `[cA_n, cD_n, cD_n-1, ..., cD2, cD1]` [list], Ordered list of coefficients arrays where `n` denotes the level of decomposition. The first element (`cA_n`) of the result is approximation coefficients array and the following elements (`cD_n - cD_1`) are details coefficients arrays.

**Examples**

```python
>>> from pywt import wavedec
>>> coeffs = wavedec([1,2,3,4,5,6,7,8], 'db1', level=2)
>>> cA2, cD2, cD1 = coeffs
>>> cD1
array([-0.70710678, -0.70710678, -0.70710678, -0.70710678])
>>> cD2
array([-2., -2.])
>>> cA2
array([ 5., 13.])
```

**Partial Discrete Wavelet Transform data decomposition downcoef**

```python
pywt.downcoef(part, data, wavelet, mode='symmetric', level=1)
```
Partial Discrete Wavelet Transform data decomposition.

Similar to `pywt.dwt`, but computes only one set of coefficients. Useful when you need only approximation or only details at the given level.

**Parameters**

- `part`: [str], Coefficients type:
  - `a` - approximations reconstruction is performed
  - `d` - details reconstruction is performed
- `data`: [array_like], Input signal.
- `wavelet`: [Wavelet object or name], Wavelet to use
- `mode`: [str, optional], Signal extension mode, see *Modes*.
- `level`: [int, optional], Decomposition level. Default is 1.

**Returns**
coeffs [ndarray] 1-D array of coefficients.

See also:

upcoef

Maximum decomposition level - dwt_max_level, dwtn_max_level

pywt.dwt_max_level(data_len, filter_len)

Compute the maximum useful level of decomposition.

Parameters

data_len [int] Input data length.

filter_len [int, str or Wavelet] The wavelet filter length. Alternatively, the name of a discrete wavelet or a Wavelet object can be specified.

Returns

max_level [int] Maximum level.

Notes

The rationale for the choice of levels is the maximum level where at least one coefficient in the output is uncorrupted by edge effects caused by signal extension. Put another way, decomposition stops when the signal becomes shorter than the FIR filter length for a given wavelet. This corresponds to:

\[
\text{max}_{\text{level}} = \left\lfloor \log_2 \left( \frac{\text{data} \_ \text{len}}{\text{filter} \_ \text{len} - 1} \right) \right\rfloor
\]

Examples

```python
>>> import pywt
>>> w = pywt.Wavelet('sym5')
>>> pywt.dwt_max_level(data_len=1000, filter_len=w.dec_len)
6
>>> pywt.dwt_max_level(1000, w)
6
>>> pywt.dwt_max_level(1000, 'sym5')
6
```

pywt.dwtn_max_level(shape, wavelet, axes=None)

Compute the maximum level of decomposition for n-dimensional data.

This returns the maximum number of levels of decomposition suitable for use with wavedec, wavedec2 or wavedecn.

Parameters

shape [sequence of ints] Input data shape.

wavelet [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in axes.

axes [sequence of ints, optional] Axes over which to compute the DWT. Axes may not be repeated.
Returns

- **level** [int] Maximum level.

Notes

The level returned is the smallest `dwt_max_level` over all axes.

Examples

```python
>>> import pywt
>>> pywt.dwt_max_level((64, 32), 'db2')
3
```

**Result coefficients length - dwt_coeff_len**

```python
pywt.dwt_coeff_len(data_len, filter_len, mode='symmetric')
```

Returns length of dwt output for given data length, filter length and mode

Parameters

- **data_len** [int] Data length.
- **filter_len** [int] Filter length.
- **mode** [str, optional] Signal extension mode, see *Modes*.

Returns

- **len** [int] Length of dwt output.

Notes

For all modes except periodization:

```python
len(cA) == len(cD) == floor((len(data) + wavelet.dec_len - 1) / 2)
```

for periodization mode (“per”):

```python
len(cA) == len(cD) == ceil(len(data) / 2)
```

Based on the given input data length (`data_len`), wavelet decomposition filter length (`filter_len`) and *signal extension mode*, the `dwt_coeff_len()` function calculates the length of the resulting coefficients arrays that would be created while performing `dwt()` transform.

*filter_len* can be either an int or Wavelet object for convenience.

### 5.2.4 Inverse Discrete Wavelet Transform (IDWT)

**Single level idwt**

```python
pywt.idwt(cA, cD, wavelet, mode='symmetric', axis=-1)
```

Single level Inverse Discrete Wavelet Transform.

Parameters
cA [array_like or None] Approximation coefficients. If None, will be set to array of zeros with same shape as cD.

cD [array_like or None] Detail coefficients. If None, will be set to array of zeros with same shape as cA.

wavelet [Wavelet object or name] Wavelet to use

mode [str, optional (default: 'symmetric')] Signal extension mode, see Modes.

axis: int, optional Axis over which to compute the inverse DWT. If not given, the last axis is used.

Returns
rec: array_like Single level reconstruction of signal from given coefficients.

Examples

```python
>>> import pywt

>>> (cA, cD) = pywt.dwt([1,2,3,4,5,6], 'db2', 'smooth')

>>> pywt.idwt(cA, cD, 'db2', 'smooth')
array([ 1.,  2.,  3.,  4.,  5.,  6.])
```

One of the neat features of idwt is that one of the cA and cD arguments can be set to None. In that situation the reconstruction will be performed using only the other one. Mathematically speaking, this is equivalent to passing a zero-filled array as one of the arguments.

```python
>>> (cA, cD) = pywt.dwt([1,2,3,4,5,6], 'db2', 'smooth')
>>> A = pywt.idwt(cA, None, 'db2', 'smooth')
>>> D = pywt.idwt(None, cD, 'db2', 'smooth')
>>> A + D
array([ 1.,  2.,  3.,  4.,  5.,  6.])
```

**Multilevel reconstruction using waverec**

**pywt.waverec** (coeffs, wavelet, mode='symmetric', axis=-1)

Multilevel 1D Inverse Discrete Wavelet Transform.

Parameters

coeffs [array_like] Coefficients list [cAn, cDn, cDn-1, ..., cD2, cD1]

wavelet [Wavelet object or name string] Wavelet to use

mode [str, optional] Signal extension mode, see Modes.

axis: int, optional Axis over which to compute the inverse DWT. If not given, the last axis is used.

Notes

It may sometimes be desired to run waverec with some sets of coefficients omitted. This can best be done by setting the corresponding arrays to zero arrays of matching shape and dtype. Explicitly removing list entries or setting them to None is not supported.

Specifically, to ignore detail coefficients at level 2, one could do:
coeffs[-2] == np.zeros_like(coeffs[-2])

**Examples**

```python
>>> import pywt
>>> coeffs = pywt.wavedec([1,2,3,4,5,6,7,8], 'db1', level=2)
>>> pywt.waverec(coeffs, 'db1')
array([ 1.,  2.,  3.,  4.,  5.,  6.,  7.,  8.])
```

**Direct reconstruction with upcoef**

`pywt.upcoef(part, coeffs, wavelet, level=1, take=0)`

Direct reconstruction from coefficients.

**Parameters**

- `part` [str] Coefficients type: * 'a' - approximations reconstruction is performed * 'd' - details reconstruction is performed
- `coeffs` [array_like] Coefficients array to recontruct
- `wavelet` [Wavelet object or name] Wavelet to use
- `level` [int, optional] Multilevel reconstruction level. Default is 1.
- `take` [int, optional] Take central part of length equal to 'take' from the result. Default is 0.

**Returns**

- `rec` [ndarray] 1-D array with reconstructed data from coefficients.

**See also:**

`downcoef`

**Examples**

```python
>>> import pywt
>>> data = [1,2,3,4,5,6]
>>> (cA, cD) = pywt.dwt(data, 'db2', 'smooth')
>>> pywt.upcoef('a', cA, 'db2') + pywt.upcoef('d', cD, 'db2')
array([-0.25 , -0.4330127 , 1. ,  2. ,  3. ,  4. ,  5. ,  6. ,  4.72161911])
>>> n = len(data)
>>> pywt.upcoef('a', cA, 'db2', take=n) + pywt.upcoef('d', cD, 'db2', take=n)
array([ 1.,  2.,  3.,  4.,  5.,  6.])
```

### 5.2.5 Overview of multilevel wavelet decompositions

There are a number of different ways a wavelet decomposition can be performed for multiresolution analysis of n-dimensional data. Here we will review the three approaches currently implemented in PyWavelets. 2D cases are illustrated, but each of the approaches extends to the n-dimensional case in a straightforward manner.
**Multilevel Discrete Wavelet Transform**

The most common approach to the multilevel discrete wavelet transform involves further decomposition of only the approximation subband at each subsequent level. This is also sometimes referred to as the Mallat decomposition [Mall89]. In 2D, the discrete wavelet transform produces four sets of coefficients corresponding to the four possible compinations of the wavelet decomposition filters over the two separate axes. (In n-dimensions, there are \(2^n\) sets of coefficients). For subsequent levels of decomposition, only the approximation coefficients (the lowpass subband) are further decomposed.

In PyWavelets, this decomposition is implemented for n-dimensional data by `wavedecn()` and the inverse by `waverecn()`. 1D and 2D versions of these routines also exist. It is illustrated in the figure below. The top row indicates the coefficient names as used by `wavedec2()` after each level of decomposition. The bottom row shows wavelet coefficients for the cameraman image (with each subband independently normalized for easier visualization).

```python
import numpy as np
import pywt
from matplotlib import pyplot as plt
from pywt._doc_utils import wavedec2_keys, draw_2d_wp_basis

x = pywt.data.camera().astype(np.float32)
shape = x.shape

max_lev = 3  # how many levels of decomposition to draw
label_levels = 3  # how many levels to explicitly label on the plots

fig, axes = plt.subplots(2, 4, figsize=[14, 8])
for level in range(0, max_lev + 1):
    if level == 0:
        # show the original image before decomposition
        axes[0, 0].set_axis_off()
        axes[1, 0].imshow(x, cmap=plt.cm.gray)
        axes[1, 0].set_title('Image')
        axes[1, 0].set_axis_off()
        continue

    # plot subband boundaries of a standard DWT basis
    draw_2d_wp_basis(shape, wavedec2_keys(level), ax=axes[0, level],
                     label_levels=label_levels)
    axes[0, level].set_title('level decomposition'.format(level))

    # compute the 2D DWT
    c = pywt.wavedec2(x, 'db2', mode='periodization', level=level)
    # normalize each coefficient array independently for better visibility
    c[0] /= np.abs(c[0]).max()
    for detail_level in range(level):
        c[detail_level + 1] = [d/np.abs(d).max() for d in c[detail_level + 1]]
        # show the normalized coefficients
        arr, slices = pywt coeff_to_array(c)
        axes[1, level].imshow(arr, cmap=plt.cm.gray)
        axes[1, level].set_title('Coefficients level'.format(level))
        axes[1, level].set_axis_off()

plt.tight_layout()
plt.show()
```

It can be seen that many of the coefficients are near zero (gray). This ability of the wavelet transform to sparsely represent natural images is a key property that makes it desirable in applications such as image compression and restoration.
Fully Separable Discrete Wavelet Transform

An alternative decomposition results in first fully decomposing one axis of the data prior to moving onto each additional axis in turn. This is illustrated for the 2D case in the upper right panel of the figure below. This approach has a factor of two higher computational cost as compared to the Mallat approach, but has advantages in compactly representing anisotropic data. A demo of this is available).

This form of the DWT is also sometimes referred to as the tensor wavelet transform or the hyperbolic wavelet transform. In PyWavelets it is implemented for n-dimensional data by fswavedecn() and the inverse by fswaverecn().

Wavelet Packet Transform

Another possible choice is to apply additional levels of decomposition to all wavelet subbands from the first level as opposed to only the approximation subband. This is known as the wavelet packet transform and is illustrated in 2D in the lower left panel of the figure. It is also possible to only perform any subset of the decompositions, resulting in a wide number of potential wavelet packet bases. An arbitrary example is shown in the lower right panel of the figure below.

A further description is available in the wavelet packet documentation.

For the wavelet packets, the plots below use “natural” ordering for simplicity, but this does not directly match the “frequency” ordering for these wavelet packets. It is possible to rearrange the coefficients into frequency ordering (see the get_level method of WaveletPacket2D and [Wick94] for more details).

from itertools import product
import numpy as np
from matplotlib import pyplot as plt
from pywt._doc_utils import (wavedec_keys, wavedec2_keys, draw_2d_wp_basis,
                          draw_2d_fswavedecn_basis)
shape = (512, 512)

max_lev = 4  # how many levels of decomposition to draw
label_levels = 2  # how many levels to explicitly label on the plots

if False:
    fig, axes = plt.subplots(1, 4, figsize=[16, 4])
    axes = axes.ravel()
else:
    fig, axes = plt.subplots(2, 2, figsize=[8, 8])
    axes = axes.ravel()

# plot a 5-level standard DWT basis
draw_2d_wp_basis(shape, wavedec2_keys(max_lev), ax=axes[0],
                 label_levels=label_levels)
axes[0].set_title('wavedec2 ({} level)'.format(max_lev))

# plot for the fully separable case
draw_2d_fswavedecn_basis(shape, max_lev, ax=axes[1], label_levels=label_levels)
axes[1].set_title('fswavedecn ({} level)'.format(max_lev))

# get all keys corresponding to a full wavelet packet decomposition
wp_keys = list(product(['a', 'd', 'h', 'v'], repeat=max_lev))
draw_2d_wp_basis(shape, wp_keys, ax=axes[2])
axes[2].set_title('wavelet packet
(full: {} level)'.format(max_lev))

# plot an example of a custom wavelet packet basis
keys = ['aaaa', 'aad', 'aaah', 'aaav', 'aad', 'aah', 'aava', 'aad', 'aah', 'aavh',
        'aavv', 'ad', 'ah', 'ava', 'avd', 'avh', 'avv', 'd', 'h',
        'vaa', 'vad', 'vah', 'vav', 'vd', 'vh', 'vv']
draw_2d_wp_basis(shape, keys, ax=axes[3], label_levels=label_levels)
axes[3].set_title('wavelet packet\n(custom)'.format(max_lev))

plt.tight_layout()
plt.show()

References

5.2.6 2D Forward and Inverse Discrete Wavelet Transform

Single level dwt2

pywt.dwt2(data, wavelet, mode='symmetric', axes=(-2, -1))

2D Discrete Wavelet Transform.

Parameters

data [array_like] 2D array with input data

wavelet [Wavelet object or name string, or 2-tuple of wavelets] Wavelet to use. This can also
be a tuple containing a wavelet to apply along each axis in axes.

mode [str or 2-tuple of strings, optional] Signal extension mode, see Modes. This can also be a
tuple of modes specifying the mode to use on each axis in axes.

axes [2-tuple of ints, optional] Axes over which to compute the DWT. Repeated elements mean
the DWT will be performed multiple times along these axes.
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wavedec2 (4 level)

fswaveedcn (4 level)

wavelet packet
(full: 4 level)

wavelet packet
(custom)
Returns

(cA, (cH, cV, cD)) [tuple] Approximation, horizontal detail, vertical detail and diagonal detail coefficients respectively. Horizontal refers to array axis 0 (or axes[0] for user-specified axes).

Examples

```python
>>> import numpy as np
>>> import pywt

>>> data = np.ones((4,4), dtype=np.float64)
>>> coeffs = pywt.dwt2(data, 'haar')
>>> cA, (cH, cV, cD) = coeffs
>>> cA
array([[ 2., 2.],
       [ 2., 2.]])
>>> cV
array([[ 0., 0.],
       [ 0., 0.]])
```

The relation to the other common data layout where all the approximation and details coefficients are stored in one big 2D array is as follows:

```
---------
| cA(LL) |
| cH(LH) |
---------
| cV(HL) |
| cD(HH) |
```

The relation to the other common data layout where all the approximation and details coefficients are stored in one big 2D array is as follows:

PyWavelets does not follow this pattern because of pure practical reasons of simple access to particular type of the output coefficients.

**Single level idwt2**

pywt.idwt2 (coeffs, wavelet, mode='symmetric', axes=(-2,-1))

2-D Inverse Discrete Wavelet Transform.

Reconstructs data from coefficient arrays.

Parameters

coeffs [tuple] (cA, (cH, cV, cD)) A tuple with approximation coefficients and three details coefficients 2D arrays like from dwt2. If any of these components are set to None, it will be treated as zeros.

wavelet [Wavelet object or name string, or 2-tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in axes.

mode [str or 2-tuple of strings, optional] Signal extension mode, see Modes. This can also be a tuple of modes specifying the mode to use on each axis in axes.

axes [2-tuple of ints, optional] Axes over which to compute the IDWT. Repeated elements mean the IDWT will be performed multiple times along these axes.
Examples

```python
>>> import numpy as np
>>> import pywt
>>> data = np.array([[1, 2], [3, 4]], dtype=np.float64)
>>> coeffs = pywt.dwt2(data, 'haar')
>>> pywt.idwt2(coeffs, 'haar')
aarray([[ 1.,  2.],
        [ 3.,  4.]])
```

2D multilevel decomposition using `wavedec2`

```python
pywt.wavedec2(data, wavelet, mode='symmetric', level=None, axes=(-2, -1))
```

Multilevel 2D Discrete Wavelet Transform.

Parameters

- `data` [ndarray] 2D input data
- `wavelet` [Wavelet object or name string, or 2-tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in `axes`.
- `mode` [str or 2-tuple of str, optional] Signal extension mode, see `Modes`. This can also be a tuple containing a mode to apply along each axis in `axes`.
- `level` [int, optional] Decomposition level (must be >= 0). If level is None (default) then it will be calculated using the `dwt_max_level` function.
- `axes` [2-tuple of ints, optional] Axes over which to compute the DWT. Repeated elements are not allowed.

Returns

- `[cAn, (cHn, cVn, cDn), ..., (cH1, cV1, cD1)]` [list] Coefficients list. For user-specified `axes`, `cH*` corresponds to `axes[0]` while `cV*` corresponds to `axes[1]`. The first element returned is the approximation coefficients for the nth level of decomposition. Remaining elements are tuples of detail coefficients in descending order of decomposition level. (i.e. `cH1` are the horizontal detail coefficients at the first level)

Examples

```python
>>> import pywt
>>> import numpy as np
>>> coeffs = pywt.wavedec2(np.ones((4,4)), 'db1')
>>> # Levels:
>>> len(coeffs)-1
2
>>> pywt.waverec2(coeffs, 'db1')
aarray([[ 1.,  1.,  1.,  1.],
        [ 1.,  1.,  1.,  1.],
        [ 1.,  1.,  1.,  1.],
        [ 1.,  1.,  1.,  1.]])
```
**2D multilevel reconstruction using waverec2**

`pywt.waverec2(coeffs, wavelet, mode='symmetric', axes=(-2, -1))`

Multilevel 2D Inverse Discrete Wavelet Transform.

- **coeffs** [list or tuple] Coefficients list `[cAn, (cHn, cVn, cDn), ... (cH1, cV1, cD1)]`
- **wavelet** [Wavelet object or name string, or 2-tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in `axes`.
- **mode** [str or 2-tuple of str, optional] Signal extension mode, see `Modes`. This can also be a tuple containing a mode to apply along each axis in `axes`.
- **axes** [2-tuple of ints, optional] Axes over which to compute the IDWT. Repeated elements are not allowed.

**Returns**

2D array of reconstructed data.

**Notes**

It may sometimes be desired to run `waverec2` with some sets of coefficients omitted. This can best be done by setting the corresponding arrays to zero arrays of matching shape and dtype. Explicitly removing list or tuple entries or setting them to None is not supported.

Specifically, to ignore all detail coefficients at level 2, one could do:

```python
coeffs[-2] == tuple([np.zeros_like(v) for v in coeffs[-2]])
```

**Examples**

```python
>>> import pywt
>>> import numpy as np
>>> coeffs = pywt.wavedec2(np.ones((4,4)), 'db1')
>>> # Levels:
>>> len(coeffs)-1
2
>>> pywt.waverec2(coeffs, 'db1')
array([[ 1.,  1.,  1.,  1.],
       [ 1.,  1.,  1.,  1.],
       [ 1.,  1.,  1.,  1.],
       [ 1.,  1.,  1.,  1.]]
```

**2D coordinate conventions**

The labels for “horizontal” and “vertical” used by `dwt2` and `idwt2` follow the common mathematical convention that coordinate axis 0 is horizontal while axis 1 is vertical:

```plaintext
dwt2, idwt2 convention
------------------------
axis 1 ^
    |   
    |   
    |   
(continues on next page)
```
Note that this is different from another common convention used in computer graphics and image processing (e.g. by matplotlib’s `imshow` and functions in `scikit-image`). In those packages axis 0 is a vertical axis and axis 1 is horizontal as follows:

```plaintext
|-------->  
axis 0
```

### 5.2.7 nD Forward and Inverse Discrete Wavelet Transform

#### _multilevel_

Multilevel 1D and 2D Discrete Wavelet Transform and Inverse Discrete Wavelet Transform.

#### Single level - dwtn

`pywt.dwtn(data, wavelet, mode='symmetric', axes=None)`

Single-level n-dimensional Discrete Wavelet Transform.

**Parameters**

- `data` [array_like] n-dimensional array with input data.
- `wavelet` [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in `axes`.
- `mode` [str or tuple of string, optional] Signal extension mode used in the decomposition, see *Modes*. This can also be a tuple of modes specifying the mode to use on each axis in `axes`.
- `axes` [sequence of ints, optional] Axes over which to compute the DWT. Repeated elements mean the DWT will be performed multiple times along these axes. A value of `None` (the default) selects all axes.

Axes may be repeated, but information about the original size may be lost if it is not divisible by $2^\text{nrepeats}$. The reconstruction will be larger, with additional values derived according to the `mode` parameter. `pywt.wavedecn` should be used for multilevel decomposition.

**Returns**

- `coeffs` [dict] Results are arranged in a dictionary, where key specifies the transform type on each dimension and value is a n-dimensional coefficients array.

For example, for a 2D case the result will look something like this:

```plaintext
{'aa': <coeffs>  # A(LL) - approx. on 1st dim, approx. on 2nd dim
'ad': <coeffs>  # V(LH) - approx. on 1st dim, det. on 2nd dim
'da': <coeffs>  # H(HL) - det. on 1st dim, approx. on 2nd dim
```
For user-specified `axes`, the order of the characters in the dictionary keys map to the specified `axes`.

### Single level - `idwt`

`pywt.idwt(coeffs, wavelet, mode='symmetric', axes=None)`

Single-level n-dimensional Inverse Discrete Wavelet Transform.

**Parameters**

- `coeffs`: dict Dictionary as in output of `dwt`. Missing or `None` items will be treated as zeros.
- `wavelet`: [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in `axes`.
- `mode`: [str or list of string, optional] Signal extension mode used in the decomposition, see `Modes`. This can also be a tuple of modes specifying the mode to use on each axis in `axes`.
- `axes`: [sequence of ints, optional] Axes over which to compute the IDWT. Repeated elements mean the IDWT will be performed multiple times along these axes. A value of `None` (the default) selects all axes.

For the most accurate reconstruction, the axes should be provided in the same order as they were provided to `dwt`.

**Returns**

- `data`: ndarray Original signal reconstructed from input data.

### Multilevel decomposition - `wavedecn`

`pywt.wavedecn(data, wavelet, mode='symmetric', level=None, axes=None)`

Multilevel nD Discrete Wavelet Transform.

**Parameters**

- `data`: [ndarray] nD input data
- `wavelet`: [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in `axes`.
- `mode`: [str or tuple of str, optional] Signal extension mode, see `Modes`. This can also be a tuple containing a mode to apply along each axis in `axes`.
- `level`: [int, optional] Decomposition level (must be >= 0). If level is None (default) then it will be calculated using the `dwt_max_level` function.
- `axes`: [sequence of ints, optional] Axes over which to compute the DWT. Axes may not be repeated. The default is None, which means transform all axes (`axes = range(data.ndim)`).

**Returns**

- `[cAn, {details_level_n}, ... {details_level_1}]` [list] Coefficients list. Coefficients are listed in descending order of decomposition level. `cAn` are the approximation coefficients at level n. Each `details_level_i` element is a dictionary containing detail coefficients at level i.
of the decomposition. As a concrete example, a 3D decomposition would have the following set of keys in each `details_level_i` dictionary:

```
{'aad', 'ada', 'daa', 'add', 'dad', 'dda', 'ddd'}
```

where the order of the characters in each key map to the specified `axes`.

### Examples

```python
>>> import numpy as np
>>> from pywt import wavedecn, waverecn

>>> coeffs = wavedecn(np.ones((4, 4, 4)), 'db1')

>>> # Levels:
>>> len(coeffs) - 1
2

>>> waverecn(coeffs, 'db1')

array([[[[1., 1., 1., 1.],
        [1., 1., 1., 1.],
        [1., 1., 1., 1.],
        [1., 1., 1., 1.]],
        [[1., 1., 1., 1.],
        [1., 1., 1., 1.],
        [1., 1., 1., 1.],
        [1., 1., 1., 1.]],
        [[1., 1., 1., 1.],
        [1., 1., 1., 1.],
        [1., 1., 1., 1.],
        [1., 1., 1., 1.]],
        [[1., 1., 1., 1.],
        [1., 1., 1., 1.],
        [1., 1., 1., 1.],
        [1., 1., 1., 1.]]])
```

**Multilevel reconstruction - waverecn**

```
pywt.waverecn(coeffs, wavelet, mode='symmetric', axes=None)
```

Multilevel nD Inverse Discrete Wavelet Transform.

- **coeffs** [array_like] Coefficients list [cAn, {details_level_n}, . . . {details_level_1}]
- **wavelet** [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in `axes`.
- **mode** [str or tuple of str, optional] Signal extension mode, see `Modes`. This can also be a tuple containing a mode to apply along each axis in `axes`.
- **axes** [sequence of ints, optional] Axes over which to compute the IDWT. Axes may not be repeated.

**Returns**

nD array of reconstructed data.

**Notes**

It may sometimes be desired to run `waverecn` with some sets of coefficients omitted. This can best be done by setting the corresponding arrays to zero arrays of matching shape and dtype. Explicitly removing list or
dictionary entries or setting them to None is not supported.

Specifically, to ignore all detail coefficients at level 2, one could do:

```python
coeffs[-2] = {k: np.zeros_like(v) for k, v in coeffs[-2].items()}
```

Examples

```python
>>> import numpy as np
>>> from pywt import wavedecn, waverecn
>>> coeffs = wavedecn(np.ones((4, 4, 4)), 'db1')
>>> # Levels:
>>> len(coeffs)-1
2
>>> waverecn(coeffs, 'db1')  # doctest: +NORMALIZE_WHITESPACE
array([[[ 1., 1., 1., 1.],
        [ 1., 1., 1., 1.],
        [ 1., 1., 1., 1.],
        [ 1., 1., 1., 1.]],
       [[ 1., 1., 1., 1.],
        [ 1., 1., 1., 1.],
        [ 1., 1., 1., 1.],
        [ 1., 1., 1., 1.]],
       [[ 1., 1., 1., 1.],
        [ 1., 1., 1., 1.],
        [ 1., 1., 1., 1.],
        [ 1., 1., 1., 1.]],
       [[ 1., 1., 1., 1.],
        [ 1., 1., 1., 1.],
        [ 1., 1., 1., 1.],
        [ 1., 1., 1., 1.]]])
```

Multilevel fully separable decomposition - `fswavedecn`

`pywt.fswavedecn` *(data, wavelet, mode='symmetric', levels=None, axes=None)*

Fully Separable Wavelet Decomposition.

This is a variant of the multilevel discrete wavelet transform where all levels of decomposition are performed along a single axis prior to moving onto the next axis. Unlike in `wavedecn`, the number of levels of decomposition are not required to be the same along each axis which can be a benefit for anisotropic data.

Parameters

- **data**: array_like  Input data
- **wavelet**: [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in axes.
- **mode**: [str or tuple of str, optional] Signal extension mode, see Modes. This can also be a tuple containing a mode to apply along each axis in axes.
- **levels**: [int or sequence of ints, optional] Decomposition levels along each axis (must be &ge; 0).
  If an integer is provided, the same number of levels are used for all axes. If levels is None (default), `dwt_max_level` will be used to compute the maximum number of levels possible for each axis.
- **axes**: [sequence of ints, optional] Axes over which to compute the transform. Axes may not be repeated. The default is to transform along all axes.
Returns

`fswavedecn_result` [FswavedecnResult object] Contains the wavelet coefficients, slice objects to allow obtaining the coefficients per detail or approximation level, and more. See `FswavedecnResult` for details.

See also:

`fswaverecn` inverse of `fswavedecn`

Notes

This transformation has been variously referred to as the (fully) separable wavelet transform (e.g. refs [1], [3]), the tensor-product wavelet ([2]) or the hyperbolic wavelet transform ([4]). It is well suited to data with anisotropic smoothness.

In [2] it was demonstrated that fully separable transform performs at least as well as the DWT for image compression. Computation time is a factor 2 larger than that for the DWT.

References

[1], [2], [3], [4]

Examples

```python
>>> from pywt import fswavedecn
>>> fs_result = fswavedecn(np.ones((32, 32)), 'sym2', levels=(1, 3))
>>> print(fs_result.detail_keys())
[(0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3)]
>>> approx_coeffs = fs_result.approx
>>> detail_1_2 = fs_result[(1, 2)]
```

Multilevel fully separable reconstruction - `fswaverecn`

`pywt.fswaverecn(fswavedecn_result)`

Fully Separable Inverse Wavelet Reconstruction.

Parameters

`fswavedecn_result` [FswavedecnResult object] FswavedecnResult object from `fswavedecn`.

Returns

`reconstructed` [ndarray] Array of reconstructed data.

See also:

`fswavedecn` inverse of `fswaverecn`

Notes

This transformation has been variously referred to as the (fully) separable wavelet transform (e.g. refs [1], [3]), the tensor-product wavelet ([2]) or the hyperbolic wavelet transform ([4]). It is well suited to data with anisotropic smoothness.
In [2] it was demonstrated that the fully separable transform performs at least as well as the DWT for image compression. Computation time is a factor 2 larger than that for the DWT.

References

[1], [2], [3], [4]

Multilevel fully separable reconstruction coeffs - FswavedecnResult

class pywt.FswavedecnResult(coeffs, coeff_slices, wavelets, mode Enums, axes)

Object representing fully separable wavelet transform coefficients.

Parameters

coeffs [ndarray] The coefficient array.

coeff_slices [list] List of slices corresponding to each detail or approximation coefficient array.

wavelets [list of pywt.DiscreteWavelet objects] The wavelets used. Will be a list with length

equal to len(axes).

mode Enums [list of int] The border modes used. Will be a list with length equal to

len(axes).

axes [tuple of int] The set of axes over which the transform was performed.

Attributes

approx ndarray: The approximation coefficients.

axes List of str: The axes the transform was performed along.

coeff_slices List: List of coefficient slices.

coeffs ndarray: All coefficients stacked into a single array.

levels List of int: Levels of decomposition along each transformed axis.

modes List of str: The border mode used along each transformed axis.

ndim int: Number of data dimensions.

ndim_transform int: Number of axes transformed.

wavelet_names List of pywt.DiscreteWavelet: wavelet for each transformed axis.

wavelets List of pywt.DiscreteWavelet: wavelet for each transformed axis.

Methods

detail_keys() Return a list of all detail coefficient keys.

5.2.8 Handling DWT Coefficients

Convenience routines are available for converting the outputs of the multilevel dwt functions (wavedec, wavedec2 and wavedecn) to and from a single, concatenated coefficient array.
Concatenating all coefficients into a single n-d array

```py
pywt.coeffs_to_array(coeffs, padding=0, axes=None)
```

Arrange a wavelet coefficient list from `wavedecn` into a single array.

**Parameters**

- `coeffs` [array-like] Dictionary of wavelet coefficients as returned by `pywt.wavedecn`
- `padding` [float or None, optional] The value to use for the background if the coefficients cannot be tightly packed. If None, raise an error if the coefficients cannot be tightly packed.
- `axes` [sequence of ints, optional] Axes over which the DWT that created `coeffs` was performed. The default value of None corresponds to all axes.

**Returns**

- `coeff_slices` [list] List of slices corresponding to each coefficient. As a 2D example,
  
  ```python
  coeff_arr[coeff_slices[1]['dd']]
  ```

  would extract the first level detail coefficients from `coeff_arr`.

**See also:**

- `array_to_coeffs` the inverse of `coeffs_to_array`

**Notes**

Assume a 2D coefficient dictionary, `c`, from a two-level transform.

Then all 2D coefficients will be stacked into a single larger 2D array as follows:

```
+---------------+---------------+-------------------------------+
| | | |
| c[0] | c[1]['da'] | |
| | | |
| +---------------+---------------+ c[2]['da'] |
| | | |
| c[1]['ad'] | c[1]['dd'] | |
| | | |
| +-------------------------------+-------------------------------+
| | |
| | |
| | |
| c[2]['ad'] | c[2]['dd'] |
| | |
| | |
| +-------------------------------+-------------------------------+
```

If the transform was not performed with mode “periodization” or the signal length was not a multiple of `2**level`, coefficients at each subsequent scale will not be exactly 1/2 the size of those at the previous level due to additional coefficients retained to handle the boundary condition. In these cases, the default setting of `padding=0` indicates to pad the individual coefficient arrays with 0 as needed so that they can be stacked into a single, contiguous array.
Examples

```python
>>> import pywt
>>> cam = pywt.data.camera()
>>>coeffs = pywt.wavedecn(cam, wavelet='db2', level=3)
>>>arr, coeff_slices = pywt.coeffs_to_array(coeffs)
```

Splitting concatenated coefficient array back into its components

pywt.array_to_coeffs(arr, coeff_slices, output_format='wavedecn')

Convert a combined array of coefficients back to a list compatible with waverecn.

**Parameters**

- **arr** [array-like] An array containing all wavelet coefficients. This should have been generated via coeffs_to_array.
- **coeff_slices** [list of tuples] List of slices corresponding to each coefficient as obtained from array_to_coeffs.
- **output_format** [\{'wavedec', 'wavedec2', 'wavedecn'\}] Make the form of the coefficients compatible with this type of multilevel transform.

**Returns**

- **coeffs**: array-like Wavelet transform coefficient array.

**See also:**

coeffs_to_array the inverse of array_to_coeffs

**Notes**

A single large array containing all coefficients will have subsets stored, into a waverecn list, c, as indicated below:

```
+---------------+---------------+-------------------------------+
|               |               |                               |
| c[0]          | c[1][‘da’]    |                               |
|               |               |                               |
|               |               | c[2][‘da’]                    |
| c[1][‘ad’]    | c[1][‘dd’]    |                               |
|               |               |                               |
|               |               |                               |
|               |               | c[2][‘ad’]                    |
|               |               |                               |
|               |               | c[2][‘dd’]                    |
```

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Examples

```python
>>> import pywt
>>> from numpy.testing import assert_array_almost_equal
>>> cam = pywt.data.camera()
>>> coeffs = pywt.wavedecn(cam, wavelet='db2', level=3)
>>> arr, coeff_slices = pywt.coeffs_to_array(coeffs, ... output_format='wavedecn')
>>> cam_recon = pywt.waverecn(coeffs, wavelet='db2')
>>> assert_array_almost_equal(cam, cam_recon)
```

Raveling and unraveling coefficients to/from a 1D array

**pywt.ravel_coeffs**(coeffs, axes=None)
Ravel a set of multilevel wavelet coefficients into a single 1D array.

**Parameters**

- **coeffs** [array-like] A list of multilevel wavelet coefficients as returned by `wavedec`, `wavedec2` or `wavedecn`. This function is also compatible with the output of `swt`, `swt2` and `swtn` if those functions were called with `trim_approx=True`.
- **axes** [sequence of ints, optional] Axes over which the DWT that created `coeffs` was performed. The default value of None corresponds to all axes.

**Returns**

- **coeff_arr** [array-like] Wavelet transform coefficient array. All coefficients have been concatenated into a single array.
- **coeff_slices** [list] List of slices corresponding to each coefficient. As a 2D example, `coeff_arr[coeff_slices[1]['dd']]]` would extract the first level detail coefficients from `coeff_arr`.
- **coeff_shapes** [list] List of shapes corresponding to each coefficient. For example, in 2D, `coeff_shapes[1]['dd']` would contain the original shape of the first level detail coefficients array.

**See also:**

**unravel_coeffs** the inverse of ravel_coeffs

**Examples**

```python
>>> import pywt
>>> cam = pywt.data.camera()
>>> coeffs = pywt.wavedecn(cam, wavelet='db2', level=3)
>>> arr, coeff_slices, coeff_shapes = pywt.ravel_coeffs(coeffs)
```

**pywt.unravel_coeffs**(arr, coeff_slices, coeff_shapes, output_format='wavedecn')
Unravel a raveled array of multilevel wavelet coefficients.

**Parameters**

- **arr** [array-like] An array containing all wavelet coefficients. This should have been generated by applying `ravel_coeffs` to the output of `wavedec`, `wavedec2` or `wavedecn` (or via `swt`, `swt2` or `swtn` with `trim_approx=True`).

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coeff_slices  [list of tuples] List of slices corresponding to each coefficient as obtained from ravel_coeffs.

coeff_shapes  [list of tuples] List of shapes corresponding to each coefficient as obtained from ravel_coeffs.


Returns

coeffs: list List of wavelet transform coefficients. The specific format of the list elements is determined by output_format.

See also:
ravel_coeffs the inverse of unravel_coeffs

Examples

```python
>>> import pywt
>>> from numpy.testing import assert_array_almost_equal
>>> cam = pywt.data.camera()
>>> coeffs = pywt.wavedecn(cam, wavelet='db2', level=3)
>>> arr, coeff_slices, coeff_shapes = pywt.ravel_coeffs(coeffs)
>>> coeffs_from_arr = pywt.unravel_coeffs(arr, coeff_slices, coeff_shapes,
... output_format='wavedecn')
>>> cam_recon = pywt.waverecn(coeffs_from_arr, wavelet='db2')
>>> assert_array_almost_equal(cam, cam_recon)
```

Multilevel: Total size of all coefficients - wavedecn_size

pywt.wavedecn_size(shapes)
Compute the total number of wavedecn coefficients.

Parameters

shapes  [list of coefficient shapes] A set of coefficient shapes as returned by wavedecn_shapes. Alternatively, the user can specify a set of coefficients as returned by wavedecn.

Returns

size  [int] The total number of coefficients.

Examples

```python
>>> import numpy as np
>>> import pywt
>>> data_shape = (64, 32)
>>> shapes = pywt.wavedecn_shapes(data_shape, 'db2', mode='periodization')
>>> pywt.wavedecn_size(shapes)
2048
>>> coeffs = pywt.wavedecn(np.ones(data_shape), 'sym4', mode='symmetric')
(continues on next page)```
Multilevel: n-d coefficient shapes - wavedecn_shapes

\texttt{pywt.wavedecn_shapes(shape, wavelet, mode='symmetric', level=None, axes=None)}

Subband shapes for a multilevel nD discrete wavelet transform.

**Parameters**

- **shape** [sequence of ints] The shape of the data to be transformed.
- **wavelet** [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in \texttt{axes}.
- **mode** [str or tuple of str, optional] Signal extension mode, see \texttt{Modes}. This can also be a tuple containing a mode to apply along each axis in \texttt{axes}.
- **level** [int, optional] Decomposition level (must be >= 0). If level is None (default) then it will be calculated using the \texttt{dwt_max_level} function.
- **axes** [sequence of ints, optional] Axes over which to compute the DWT. Axes may not be repeated. The default is None, which means transform all axes (\texttt{axes = range(data.ndim)}).

**Returns**

- **shapes** [[cAn, {details_level_n}, \ldots {details_level_1}]] Coefficients shape list. Mirrors the output of \texttt{wavedecn}, except it contains only the shapes of the coefficient arrays rather than the arrays themselves.

**Examples**

```python
>>> import pywt

>>> pywt.wavedecn_shapes((64, 32), wavelet='db2', level=3, axes=(0, ))
[(10, 32), {'d': (10, 32)}, {'d': (18, 32)}, {'d': (33, 32)}]
```

### 5.2.9 Stationary Wavelet Transform

Stationary Wavelet Transform (SWT), also known as Undecimated wavelet transform or Algorithme à trous is a translation-invariance modification of the Discrete Wavelet Transform that does not decimate coefficients at every transformation level.

**Multilevel 1D swt**

\texttt{pywt.swt(data, wavelet, level=None, start_level=0, axis=-1, trim_approx=False, norm=False)}

Multilevel 1D stationary wavelet transform.

**Parameters**

- **data** : Input signal
- **wavelet** : Wavelet to use (Wavelet object or name)
- **level** [int, optional] The number of decomposition steps to perform.
start_level  [int, optional] The level at which the decomposition will begin (it allows one to skip a given number of transform steps and compute coefficients starting from start_level) (default: 0)

axis: int, optional  Axis over which to compute the SWT. If not given, the last axis is used.

trim_approx  [bool, optional] If True, approximation coefficients at the final level are retained.

norm  [bool, optional] If True, transform is normalized so that the energy of the coefficients will be equal to the energy of data. In other words, np.linalg.norm(data.ravel()) will equal the norm of the concatenated transform coefficients when trim_approx is True.

Returns

coeffs  [list] List of approximation and details coefficients pairs in order similar to wavedec function:

```
[(cAn, cDn), ... , (cA2, cD2), (cA1, cD1)]
```

where n equals input parameter level.

If start_level = m is given, then the beginning m steps are skipped:

```
[(cAm+n, cDm+n), ..., (cAm+1, cDm+1), (cAm, cDm)]
```

If trim_approx is True, then the output list is exactly as in pywt.wavedec, where the first coefficient in the list is the approximation coefficient at the final level and the rest are the detail coefficients:

```
[cAn, cDn, ..., cD2, cD1]
```

Notes

The implementation here follows the “algorithm a-trous” and requires that the signal length along the transformed axis be a multiple of 2**level. If this is not the case, the user should pad up to an appropriate size using a function such as numpy.pad.

A primary benefit of this transform in comparison to its decimated counterpart (pywt.wavedecn), is that it is shift-invariant. This comes at cost of redundancy in the transform (the size of the output coefficients is larger than the input).

When the following three conditions are true:

1. The wavelet is orthogonal
2. swt is called with norm=True
3. swt is called with trim_approx=True

the transform has the following additional properties that may be desirable in applications:

1. energy is conserved
2. variance is partitioned across scales

When used with norm=True, this transform is closely related to the multiple-overlap DWT (MODWT) as popularized for time-series analysis, although the underlying implementation is slightly different from the one published in [1]. Specifically, the implementation used here requires a signal that is a multiple of 2**level in length.
References

[1]

Multilevel 2D `swt2`

```
pywt.swt2(data, wavelet, level, start_level=0, axes=(-2, -1), trim_approx=False, norm=False)
```

Multilevel 2D stationary wavelet transform.

**Parameters**

- `data` [array_like] 2D array with input data
- `wavelet` [Wavelet object or name string, or 2-tuple of wavelets] Wavelet to use. This can also be a tuple of wavelets to apply per axis in `axes`.
- `level` [int] The number of decomposition steps to perform.
- `start_level` [int, optional] The level at which the decomposition will start (default: 0)
- `axes` [2-tuple of ints, optional] Axes over which to compute the SWT. Repeated elements are not allowed.
- `trim_approx` [bool, optional] If True, approximation coefficients at the final level are retained.
- `norm` [bool, optional] If True, transform is normalized so that the energy of the coefficients will be equal to the energy of `data`. In other words, `np.linalg.norm(data.ravel())` will equal the norm of the concatenated transform coefficients when `trim_approx` is True.

**Returns**

- `coeffs` [list] Approximation and details coefficients (for `start_level = m`). If `trim_approx` is `False`, approximation coefficients are retained for all levels:

```python
[
    (cA_m+level,
     (cH_m+level, cV_m+level, cD_m+level))
],
...
(cA_m+1,
    (cH_m+1, cV_m+1, cD_m+1))
],
(cA_m,
    (cH_m, cV_m, cD_m))
]
```

where `cA` is approximation, `cH` is horizontal details, `cV` is vertical details, `cD` is diagonal details and `m` is `start_level`.

If `trim_approx` is `True`, approximation coefficients are only retained at the final level of decomposition. This matches the format used by `pywt.wavedec2`:

```python
[
    cA_m+level,
    (cH_m+level, cV_m+level, cD_m+level),
    ...
    (cH_m+1, cV_m+1, cD_m+1),
]
```

(continues on next page)
Notes

The implementation here follows the “algorithm a-trous” and requires that the signal length along the transformed axes be a multiple of $2^{level}$. If this is not the case, the user should pad up to an appropriate size using a function such as `numpy.pad`.

A primary benefit of this transform in comparison to its decimated counterpart (`pywt.wavedecn`), is that it is shift-invariant. This comes at cost of redundancy in the transform (the size of the output coefficients is larger than the input).

When the following three conditions are true:

1. The wavelet is orthogonal
2. `swt2` is called with `norm=True`
3. `swt2` is called with `trim_approx=True`

the transform has the following additional properties that may be desirable in applications:

1. energy is conserved
2. variance is partitioned across scales

Multilevel n-dimensional `swtn`

`pywt.swtn(data, wavelet, level, start_level=0, axes=None, trim_approx=False, norm=False)`

n-dimensional stationary wavelet transform.

Parameters

data [array_like] n-dimensional array with input data.

wavelet [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple of wavelets to apply per axis in `axes`.

level [int] The number of decomposition steps to perform.

start_level [int, optional] The level at which the decomposition will start (default: 0)

axes [sequence of ints, optional] Axes over which to compute the SWT. A value of `None` (the default) selects all axes. Axes may not be repeated.

trim_approx [bool, optional] If True, approximation coefficients at the final level are retained.

norm [bool, optional] If True, transform is normalized so that the energy of the coefficients will be equal to the energy of `data`. In other words, `np.linalg.norm(data.ravel())` will equal the norm of the concatenated transform coefficients when `trim_approx` is True.

Returns

`[[coeffs_level_n], ..., [coeffs_level_1]]`: list of dict

Results for each level are arranged in a dictionary, where the key specifies the transform type on each dimension and value is a n-dimensional coefficients array.

For example, for a 2D case the result at a given level will look something like this:
{'aa': <coeffs>  # A(LL) - approx. on 1st dim, approx. on 2nd dim
'ad': <coeffs>  # V(LH) - approx. on 1st dim, det. on 2nd dim
'da': <coeffs>  # H(HL) - det. on 1st dim, approx. on 2nd dim
'dd': <coeffs>  # D(HH) - det. on 1st dim, det. on 2nd dim
}

For user-specified axes, the order of the characters in the dictionary keys map to the specified axes.

If trim_approx is True, the first element of the list contains the array of approximation coefficients from the final level of decomposition, while the remaining coefficient dictionaries contain only detail coefficients. This matches the behavior of pywt.wavedecn.

Notes

The implementation here follows the “algorithm a-trous” and requires that the signal length along the transformed axes be a multiple of 2**level. If this is not the case, the user should pad up to an appropriate size using a function such as numpy.pad.

A primary benefit of this transform in comparison to its decimated counterpart (pywt.wavedecn), is that it is shift-invariant. This comes at cost of redundancy in the transform (the size of the output coefficients is larger than the input).

When the following three conditions are true:

1. The wavelet is orthogonal
2. swtn is called with norm=True
3. swtn is called with trim_approx=True

the transform has the following additional properties that may be desirable in applications:

1. energy is conserved
2. variance is partitioned across scales

Maximum decomposition level - swt_max_level

pywt.swt_max_level(input_len)
Calculates the maximum level of Stationary Wavelet Transform for data of given length.

Parameters

input_len [int] Input data length.

Returns

max_level [int] Maximum level of Stationary Wavelet Transform for data of given length.

Notes

For the current implementation of the stationary wavelet transform, this corresponds to the number of times input_len is evenly divisible by two. In other words, for an n-level transform, the signal length must be a multiple of 2**n. numpy.pad can be used to pad a signal up to an appropriate length as needed.
5.2.10 Inverse Stationary Wavelet Transform

Inverse stationary wavelet transforms are provided.

**Note:** These inverse transforms are not yet optimized for speed. Only, the n-dimensional inverse transform currently has axes support.

**Multilevel 1D iswt**

pywt.iswt(coeffs, wavelet, norm=False, axis=-1)

Multilevel 1D inverse discrete stationary wavelet transform.

**Parameters**

coeffs [array_like] Coefficients list of tuples:

```
[(cAn, cDn), ..., (cA2, cD2), (cA1, cD1)]
```

where cA is approximation, cD is details. Index 1 corresponds to start_level from pywt.swt.

wavelet [Wavelet object or name string] Wavelet to use

norm [bool, optional] Controls the normalization used by the inverse transform. This must be set equal to the value that was used by pywt.swt to preserve the energy of a round-trip transform.

**Returns**

1D array of reconstructed data.

**Examples**

```python
>>> import pywt

>>> coeffs = pywt.swt([1,2,3,4,5,6,7,8], 'db2', level=2)

>>> pywt.iswt(coeffs, 'db2')
array([ 1.,  2.,  3.,  4.,  5.,  6.,  7.,  8.])
```

**Multilevel 2D iswt2**

pywt.iswt2(coeffs, wavelet, norm=False, axes=(-2, -1))

Multilevel 2D inverse discrete stationary wavelet transform.

**Parameters**

coeffs [list] Approximation and details coefficients:

```
[  
  (cA_n,  
    (cH_n, cV_n, cD_n)  
  ),  
  ...,  
  (cA_2,  
    (cH_2, cV_2, cD_2)  
  ),  
  (cA_1,  
```

(continues on next page)
where cA is approximation, cH is horizontal details, cV is vertical details, cD is diagonal details and n is the number of levels. Index 1 corresponds to `start_level` from `pywt.swt2`.

**wavelet** [Wavelet object or name string, or 2-tuple of wavelets] Wavelet to use. This can also be a 2-tuple of wavelets to apply per axis.

**norm** [bool, optional] Controls the normalization used by the inverse transform. This must be set equal to the value that was used by `pywt.swt2` to preserve the energy of a round-trip transform.

**Returns**

2D array of reconstructed data.

**Examples**

```
>>> import pywt
>>> coeffs = pywt.swt2([[1,2,3,4],[5,6,7,8],
... [9,10,11,12],[13,14,15,16]],
... 'db1', level=2)
>>> pywt.iswt2(coeffs, 'db1')
array([[ 1.,  2.,  3.,  4.],
       [ 5.,  6.,  7.,  8.],
       [ 9., 10., 11., 12.],
       [13., 14., 15., 16.]])
```

**Multilevel n-dimensional iswt**

```
pywt.iswtn(coeffs, wavelet, axes=None, norm=False)
```

Multilevel nD inverse discrete stationary wavelet transform.

**Parameters**

- **coeffs** [list] `[[coeffs_level_n], ..., [coeffs_level_1]]`: list of dict
  - **wavelet** [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple of wavelets to apply per axis in `axes`.
  - **axes** [sequence of ints, optional] Axes over which to compute the inverse SWT. Axes may not be repeated. The default is `None`, which means transform all axes (`axes = range(data.ndim)`).
  - **norm** [bool, optional] Controls the normalization used by the inverse transform. This must be set equal to the value that was used by `pywt.swt` to preserve the energy of a round-trip transform.

**Returns**

nD array of reconstructed data.
Examples

```python
>>> import pywt
>>> coeffs = pywt.swtn([[1,2,3,4],[5,6,7,8],
...                      [9,10,11,12],[13,14,15,16]],
...                      'db1', level=2)
>>> pywt.iswtn(coeffs, 'db1')
array([[ 1.,  2.,  3.,  4.],
       [ 5.,  6.,  7.,  8.],
       [ 9., 10., 11., 12.],
       [13., 14., 15., 16.]])
```

5.2.11 Multiresolution Analysis

The functions in this module can be used to project a signal onto wavelet subspaces and an approximation subspace. This is an additive decomposition such that the sum of the coefficients equals the original signal. The projected signal coefficients remains temporally aligned with the original, regardless of the symmetry of the wavelet used for the analysis.

Multilevel 1D mra

```python
pywt.mra(data, wavelet, level=None, axis=-1, transform='swt', mode='periodization')
```
Forward 1D multiresolution analysis.

It is a projection onto the wavelet subspaces.

Parameters

- **data**: array_like Input data
- **wavelet**: [Wavelet object or name string] Wavelet to use
- **level**: [int, optional] Decomposition level (must be >= 0). If level is None (default) then it will be calculated using the `dwt_max_level` function.
- **axis**: int, optional Axis over which to compute the DWT. If not given, the last axis is used. Currently only available when `transform='dwt'`.
- **transform**: [{'dwt', 'swt'}] Whether to use the DWT or SWT for the transforms.
- **mode**: [str, optional] Signal extension mode, see Modes (default: ‘symmetric’). This option is only used when transform='dwt'.

Returns

- **[cAn, {details_level_n}, ... {details_level_1}]** [list] For more information, see the detailed description in `wavedec`

See also:

- `imra`, `swt`

Notes

This is sometimes referred to as an additive decomposition because the inverse transform (``imra``) is just the sum of the coefficient arrays [1]. The decomposition using `transform='dwt'` corresponds to section 2.2 while that using an undecimated transform (`transform='swt'`) is described in section 3.2 and appendix A.
This transform does not share the variance partition property of $\text{swt}$ with $\text{norm}=\text{True}$. It does however, result in coefficients that are temporally aligned regardless of the symmetry of the wavelet used.

The redundancy of this transform is $(\text{level} + 1)$.

**References**

[1]

### Multilevel 2D mra2

$\text{pywt.mra2}(\text{data}, \text{wavelet}, \text{level}=\text{None}, \text{axes}=(-2, -1), \text{transform}='\text{swt2}', \text{mode}='\text{periodization}')$

Forward 2D multiresolution analysis.

It is a projection onto wavelet subspaces.

**Parameters**

- **data**: array_like  Input data
- **wavelet**: [Wavelet object or name string, or 2-tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in $\text{axes}$.
- **level**: [int, optional] Decomposition level (must be $\geq 0$). If level is None (default) then it will be calculated using the $\text{dwt_max_level}$ function.
- **axes**: [2-tuple of ints, optional] Axes over which to compute the DWT. Repeated elements are not allowed. Currently only available when $\text{transform}='\text{dwt2}'$.
- **transform**: [\{'dwt2', 'swt2'\}] Whether to use the DWT or SWT for the transforms.
- **mode**: [str or 2-tuple of str, optional] Signal extension mode, see $\text{Modes}$ (default: 'symmetric'). This option is only used when $\text{transform}='\text{dwt2}'$.

**Returns**

- **coeffs**: [list] For more information, see the detailed description in $\text{wavedec2}$

**See also**:

$\text{imra2}, \text{swt2}$

**Notes**

This is sometimes referred to as an additive decomposition because the inverse transform ($\text{imra2}$) is just the sum of the coefficient arrays [1]. The decomposition using $\text{transform}='\text{dwt}'$ corresponds to section 2.2 while that using an undecimated transform ($\text{transform}='\text{swt}'$) is described in section 3.2 and appendix A.

This transform does not share the variance partition property of $\text{swt2}$ with $\text{norm}=\text{True}$. It does however, result in coefficients that are temporally aligned regardless of the symmetry of the wavelet used.

The redundancy of this transform is $3 \ast \text{level} + 1$.

**References**

[1]
Multilevel n-dimensional \texttt{mran}

\texttt{pywt.mran(data, wavelet, level=None, axes=None, transform='swtn', mode='periodization')}

Forward nD multiresolution analysis.

It is a projection onto the wavelet subspaces.

**Parameters**

- \texttt{data: array_like} Input data
- \texttt{wavelet} [Wavelet object or name string, or tuple of wavelets] Wavelet to use. This can also be a tuple containing a wavelet to apply along each axis in \texttt{axes}.
- \texttt{level} [int, optional] Decomposition level (must be >= 0). If level is None (default) then it will be calculated using the \texttt{dwt_max_level} function.
- \texttt{axes} [tuple of ints, optional] Axes over which to compute the DWT. Repeated elements are not allowed.
- \texttt{transform} [{'dwtn', 'swtn'}] Whether to use the DWT or SWT for the transforms.
- \texttt{mode} [str or tuple of str, optional] Signal extension mode, see \texttt{Modes} (default: 'symmetric'). This option is only used when \texttt{transform='dwt'}.

**Returns**

- \texttt{coeffs} [list] For more information, see the detailed description in \texttt{wavedecn}.

See also:

\texttt{imran, swtn}

**Notes**

This is sometimes referred to as an additive decomposition because the inverse transform (\texttt{imran}) is just the sum of the coefficient arrays [1]. The decomposition using \texttt{transform='dwt'} corresponds to section 2.2 while that using an undecimated transform (\texttt{transform='swt'}) is described in section 3.2 and appendix A.

This transform does not share the variance partition property of \texttt{swtn} with \texttt{norm=True}. It does however, result in coefficients that are temporally aligned regardless of the symmetry of the wavelet used.

The redundancy of this transform is \((2**n - 1) * \text{level} + 1\) where \(n\) corresponds to the number of axes transformed.

**References**

[1]

**Inverse Multilevel 1D \texttt{imra}**

\texttt{pywt.imra(mra_coeffs)}

Inverse 1D multiresolution analysis via summation.

**Parameters**

- \texttt{mra_coeffs} [list of ndarray] Multiresolution analysis coefficients as returned by \texttt{mra}.

**Returns**
rec  [ndarray] The reconstructed signal.

See also:

mra

References

[1]

Inverse Multilevel 2D \texttt{imra2}

\texttt{pywt.imra2(mra_coeffs)}
Inverse 2D multiresolution analysis via summation.

Parameters

\texttt{mra_coeffs}  [list] Multiresolution analysis coefficients as returned by \texttt{mra2}.

Returns

\texttt{rec}  [ndarray] The reconstructed signal.

See also:

\texttt{mra2}

References

[1]

Inverse Multilevel n-dimensional \texttt{imran}

\texttt{pywt.imran(mra_coeffs)}
Inverse nD multiresolution analysis via summation.

Parameters

\texttt{mra_coeffs}  [list] Multiresolution analysis coefficients as returned by \texttt{mra2}.

Returns

\texttt{rec}  [ndarray] The reconstructed signal.

See also:

\texttt{mran}

References

[1]
5.2.12 Wavelet Packets

PyWavelets implements one-dimensional, two-dimensional and n-dimensional wavelet packet transform structures. The higher dimensional structures almost completely sharing programming interface with the one-dimensional tree structure.

In order to achieve this simplification, a new inheritance scheme was used in which a BaseNode base node class is a superclass for the Node, Node2D and NodeND classes.

The node classes are used as data wrappers and can be organized in trees (binary trees for 1D transform case, quad-trees for the 2D one and 2**N-ary trees in ND). They are also superclasses to the WaveletPacket, WaveletPacket2D and WaveletPacketND classes that are used as the decomposition tree roots and contain a couple additional methods.

Here 1D, 2D and ND refer to the number of axes of the data to be transformed. All wavelet packet objects can operate on general n-dimensional arrays, but the 1D or 2D classes apply transforms along only 1 or 2 dimensions. The ND classes allow transforms over an arbitrary number of axes of n-dimensional data.

The below diagram illustrates the inheritance tree:

- **BaseNode** - common interface for 1D and 2D nodes:
  - **Node** - data carrier node in a 1D decomposition tree
    * **WaveletPacket** - 1D decomposition tree root node
  - **Node2D** - data carrier node in a 2D decomposition tree
    * **WaveletPacket2D** - 2D decomposition tree root node
  - **NodeND** - data carrier node in a ND decomposition tree
    * **WaveletPacketND** - ND decomposition tree root node

BaseNode - a common interface of WaveletPacket, WaveletPacket2D and WaveletPacketND

```python
class pywt.BaseNode
```

**Note:** The BaseNode is a base class for Node, Node2D, and NodeND. It should not be used directly unless creating a new transformation type. It is included here to document the common interface of the node and wavelet packet transform classes.

```python
__init__(parent, data, node_name)
```

**Parameters**

- `parent` – parent node. If parent is `None` then the node is considered detached.
- `data` – The data associated with the node. An n-dimensional numeric array.
- `node_name` – a name identifying the coefficients type. See `Node.node_name` and `Node2D.node_name` for information on the accepted subnodes names.

**data**

Data associated with the node. An n-dimensional numeric array.

**parent**

Parent node. Used in tree navigation. `None` for root node.
wavelet

Wavelet used for decomposition and reconstruction. Inherited from parent node.

axes

A tuple of ints containing the axes along which the wavelet packet transform is to be applied.

mode

Signal extension mode for the dwt() (dwt2()) and idwt() (idwt2()) decomposition and reconstruction functions. Inherited from parent node.

level

Decomposition level of the current node. 0 for root (original data), 1 for the first decomposition level, etc.

path

Path string defining position of the node in the decomposition tree.

path_tuple

A version of path, but in tuple form rather than as a single string. The tuple form is easier to work with for n-dimensional transforms. The length of the tuple will be equal to the number of levels of decomposition at the current node.

node_name

Node name describing data coefficients type of the current subnode.

See Node.node_name and Node2D.node_name.

maxlevel

Maximum allowed level of decomposition. Evaluated from parent or child nodes.

is_empty

Checks if data attribute is None.

has_any_subnode

Checks if node has any subnodes (is not a leaf node).

decompose()

Performs Discrete Wavelet Transform on the data and returns transform coefficients.

reconstruct([update=False])

Performs Inverse Discrete Wavelet Transform on subnodes coefficients and returns reconstructed data for the current level.

Parameters update – If set, the data attribute will be updated with the reconstructed value.

Note: Descends to subnodes and recursively calls reconstruct() on them.

get_subnode(part[, decompose=True])

Returns subnode or None (see decomposition flag description).

Parameters

• part – Subnode name

• decompose – If True and subnode does not exist, it will be created using coefficients from the DWT decomposition of the current node.

__getitem__(path)

Used to access nodes in the decomposition tree by string path.

Parameters path – Path string composed from valid node names. See Node.node_name and Node2D.node_name for node naming convention.
Similar to `get_subnode()` method with `decompose=True`, but can access nodes on any level in the decomposition tree.

If node does not exist yet, it will be created by decomposition of its parent node.

```
__setitem__(path, data)
```

Used to set node or node’s data in the decomposition tree. Nodes are identified by string `path`.

**Parameters**

- **path** – Path string composed from valid node names. See `Node.node_name` and `Node2D.node_name` for node naming convention.
- **data** – numeric array or `BaseNode` subclass.

```
__delitem__(path)
```

Used to delete node from the decomposition tree.

**Parameters**

- **path** – Path string composed from valid node names. See `Node.node_name` and `Node2D.node_name` for node naming convention.

```
get_leaf_nodes([decompose=False])
```

Traverses through the decomposition tree and collects leaf nodes (nodes without any subnodes).

**Parameters**

- **decompose** – If `decompose` is `True`, the method will try to decompose the tree up to the maximum level.

```
walk(self, func[, args=()[, kwargs={}[, decompose=True ]]])
```

Traverses the decomposition tree and calls `func(node, *args, **kwargs)` on every node. If `func` returns `True`, descending to subnodes will continue.

**Parameters**

- **func** – callable accepting `BaseNode` as the first param and optional positional and keyword arguments:

  ```python
  func(node, *args, **kwargs)
  ```

- **decompose** – If `decompose` is `True` (default), the method will also try to decompose the tree up to the maximum level.

**Args** arguments to pass to the `func`

**Kwargs** keyword arguments to pass to the `func`

```
walk_depth(self, func[, args=()[, kwargs={}[, decompose=False ]]])
```

Similar to `walk()` but traverses the tree in depth-first order.

**Parameters**

- **func** – callable accepting `BaseNode` as the first param and optional positional and keyword arguments:

  ```python
  func(node, *args, **kwargs)
  ```

- **decompose** – If `decompose` is `True`, the method will also try to decompose the tree up to the maximum level.

**Args** arguments to pass to the `func`

**Kwargs** keyword arguments to pass to the `func`
WaveletPacket and Node

class pywt.Node(BaseNode)

    node_name
    Node name describing data coefficients type of the current subnode.

    For WaveletPacket case it is just as in dwt():
    • a - approximation coefficients
    • d - details coefficients

decompose()

    See also:
    dwt() for 1D Discrete Wavelet Transform output coefficients.

reconstruct()

    See also:
    idwt() for 1D Inverse Discrete Wavelet Transform

class pywt.WaveletPacket(Node)

    __init__(data, wavelet[, mode='symmetric', maxlevel=None, axis=-1])

    Parameters
    • data – data associated with the node. N-dimensional numeric array.
    • wavelet – Wavelet to use in the transform. This can be a name of the wavelet from the
      wavelist() list or a Wavelet object instance.
    • mode – Signal extension mode for the dwt() and idwt() decomposition and recon-
      struction functions.
    • maxlevel – Maximum allowed level of decomposition. If not specified it will be calcu-
      lated based on the wavelet and data length using pywt.dwt_max_level().
    • axis – The axis of the array that is to be transformed.

get_level(level[, order="natural", decompose=True])
    Collects nodes from the given level of decomposition.

    Parameters
    • level – Specifies decomposition level from which the nodes will be collected.
    • order – Specifies nodes order - natural (natural) or frequency (freq).
    • decompose – If set then the method will try to decompose the data up to the specified
      level.

    If nodes at the given level are missing (i.e. the tree is partially decomposed) and the decompose is set to
    False, only existing nodes will be returned.

reconstruct([update=True])
    Reconstruct data from the subnodes.
Parameters **update** – A boolean indicating whether the coefficients of the current node and its subnodes will be replaced with values from the reconstruction.

**WaveletPacket2D and Node2D**

class pywt.Node2D (BaseNode)

    node_name

    For **WaveletPacket2D** case it is just as in **dwt2()**:
    • a - approximation coefficients (LL)
    • h - horizontal detail coefficients (LH)
    • v - vertical detail coefficients (HL)
    • d - diagonal detail coefficients (HH)

    **decompose()**

    See also:
    dwt2() for 2D Discrete Wavelet Transform output coefficients.

    **reconstruct()**

    See also:
    idwt2() for 2D Inverse Discrete Wavelet Transform

    **expand_2d_path(self, path):**

class pywt.WaveletPacket2D (Node2D)

    __init__(data, wavelet[, mode='symmetric'[, maxlevel=None[, axes=(-2,-1)]]])

    Parameters
    • **data** – data associated with the node. N-dimensional numeric array.
    • **wavelet** – Wavelet to use in the transform. This can be a name of the wavelet from the wavelist() list or a Wavelet object instance.
    • **mode** – Signal extension mode for the dwt() and idwt() decomposition and reconstruction functions.
    • **maxlevel** – Maximum allowed level of decomposition. If not specified it will be calculated based on the wavelet and data length using pywt.dwt_max_level().
    • **axes** – The axes of the array that are to be transformed.

    **get_level(level[, order="natural"[, decompose=True ]])**

    Collects nodes from the given level of decomposition.

    Parameters
    • **level** – Specifies decomposition level from which the nodes will be collected.
    • **order** – Specifies nodes order - natural (natural) or frequency (freq).
• **decompose** – If set then the method will try to decompose the data up to the specified level.

If nodes at the given level are missing (i.e. the tree is partially decomposed) and the `decompose` is set to False, only existing nodes will be returned.

```python
reconstruct(update=True)
```
Reconstruct data from the subnodes.

**Parameters**

- **update** – A boolean indicating whether the coefficients of the current node and its subnodes will be replaced with values from the reconstruction.

---

**WaveletPacketND and NodeND**

```python
class pywt.NodeND(BaseNode)
```

**node_name**

For **WaveletPacketND** case it is just as in `dwt()`:  
- in 1D it has keys ‘a’ and ‘d’  
- in 2D it has keys ‘aa’, ‘ad’, ‘da’, ‘dd’  

```python
decompose()
```

**See also:**

- `dwt()` for ND Discrete Wavelet Transform output coefficients.
- `reconstruct()`

**See also:**

- `idwt()` for ND Inverse Discrete Wavelet Transform

```python
class pywt.WaveletPacketND(NodeND)
```

**__init__**(data, wavelet[, mode='symmetric'[, maxlevel=None[, axes=None ]]])

**Parameters**

- **data** – data associated with the node. N-dimensional numeric array.
- **wavelet** – Wavelet to use in the transform. This can be a name of the wavelet from the `wavelist()` list or a `Wavelet` object instance.
- **mode** – Signal extension `mode` for the `dwt()` and `idwt()` decomposition and reconstruction functions.
- **maxlevel** – Maximum allowed level of decomposition. If not specified it will be calculated based on the wavelet and data length using `pywt.dwt_max_level()`.
- **axes** – The axes of the array that are to be transformed.

```python
get_level(level[, decompose=True ])
```
Collects nodes from the given level of decomposition.
• **level** – Specifies decomposition level from which the nodes will be collected.
• **decompose** – If set then the method will try to decompose the data up to the specified level.

If nodes at the given level are missing (i.e. the tree is partially decomposed) and the decompose is set to False, only existing nodes will be returned.

```python
reconstruct(update=True)
```
Reconstruct data from the subnodes.

**Parameters**
- **update** – A boolean indicating whether the coefficients of the current node and its subnodes will be replaced with values from the reconstruction.

### 5.2.13 Continuous Wavelet Transform (CWT)

This section describes functions used to perform single continuous wavelet transforms.

#### Single level - cwt

```python
pywt.cwt(data, scales, wavelet)
```
One dimensional Continuous Wavelet Transform.

**Parameters**
- **data** [array_like] Input signal
- **scales** [array_like] The wavelet scales to use. One can use \( f = \text{scale2frequency}(\text{wavelet}, \text{scale})/\text{sampling\_period} \) to determine what physical frequency, \( f \). Here, \( f \) is in hertz when the sampling\_period is given in seconds.
- **wavelet** [Wavelet object or name] Wavelet to use
- **sampling\_period** [float] Sampling period for the frequencies output (optional). The values computed for coefs are independent of the choice of sampling\_period (i.e. scales is not scaled by the sampling period).
- **method** [\{'conv', 'fft', optional\}]

The method used to compute the CWT. Can be any of:
- **conv** uses numpy.convolve.
- **fft** uses frequency domain convolution.
- **auto** uses automatic selection based on an estimate of the computational complexity at each scale.

The **conv** method complexity is \( O(\text{len}(\text{scale}) \times \text{len}(...)) \). The **fft** method is \( O(N \times \log_2(N)) \) with \( N = \text{len}(\text{scale}) + \text{len}(\text{data}) - 1 \). It is well suited for large size signals but slightly slower than **conv** on small ones.

- **axis** [int, optional] Axis over which to compute the CWT. If not given, the last axis is used.

**Returns**
- **coefs** [array_like] Continuous wavelet transform of the input signal for the given scales and wavelet. The first axis of coefs corresponds to the scales. The remaining axes match the shape of data.
**frequencies** [array_like] If the unit of sampling period are seconds and given, then frequencies are in hertz. Otherwise, a sampling period of 1 is assumed.

**Notes**

Size of coefficients arrays depends on the length of the input array and the length of given scales.

**Examples**

```python
>>> import pywt
>>> import numpy as np
>>> import matplotlib.pyplot as plt

>>> x = np.arange(512)
>>> y = np.sin(2*np.pi*x/32)
>>> coef, freqs = pywt.cwt(y, np.arange(1, 129), 'gaus1')
>>> plt.matshow(coef)  # doctest: +SKIP
>>> plt.show()  # doctest: +SKIP
```

**Continuous Wavelet Families**

A variety of continuous wavelets have been implemented. A list of the available wavelet names compatible with `cwt` can be obtained by:

```python
wavlist = pywt.wavelist(kind='continuous')
```

**Mexican Hat Wavelet**

The mexican hat wavelet "mexh" is given by:

\[ \psi(t) = \frac{2}{\sqrt{3} \sqrt{\pi}} \exp\left(-\frac{t^2}{2}\right) \left(1 - t^2\right) \]

where the constant out front is a normalization factor so that the wavelet has unit energy.

**Morlet Wavelet**

The Morlet wavelet "morl" is given by:

\[ \psi(t) = \exp\left(-\frac{t^2}{2}\right) \cos(5t) \]
**Complex Morlet Wavelets**

The complex Morlet wavelet ("cmorB-C" with floating point values B, C) is given by:

\[ \psi(t) = \frac{1}{\sqrt{\pi B}} \exp\left(-\frac{t^2}{B}\right) \exp\left(j2\pi Ct\right) \]

where \(B\) is the bandwidth and \(C\) is the center frequency.

**Gaussian Derivative Wavelets**

The Gaussian wavelets ("gausP" where \(P\) is an integer between 1 and 8) correspond to the \(P\)th order derivatives of the function:

\[ \psi(t) = C \exp^{-t^2} \]

where \(C\) is an order-dependent normalization constant.

**Complex Gaussian Derivative Wavelets**

The complex Gaussian wavelets ("cgauP" where \(P\) is an integer between 1 and 8) correspond to the \(P\)th order derivatives of the function:

\[ \psi(t) = C \exp^{-j\pi t} \exp^{-t^2} \]

where \(C\) is an order-dependent normalization constant.

**Shannon Wavelets**

The Shannon wavelets ("shanB-C" with floating point values B and C) correspond to the following wavelets:

\[ \psi(t) = \sqrt{B} \sin\left(\frac{\pi B t}{\pi t}\right) \exp\left(j2\pi Ct\right) \]

where \(B\) is the bandwidth and \(C\) is the center frequency.

**Frequency B-Spline Wavelets**

The frequency B-spline wavelets ("fpspM-B-C" with integer \(M\) and floating point B, C) correspond to the following wavelets:

\[ \psi(t) = \sqrt{B} \left[\frac{\sin(\pi B t)}{\pi B t}\right]^M \exp\left(j2\pi Ct\right) \]

where \(M\) is the spline order, \(B\) is the bandwidth and \(C\) is the center frequency.
Choosing the scales for `cwt`

For each of the wavelets described below, the implementation in PyWavelets evaluates the wavelet function for $t$ over the range $[\text{wavelet.lower_bound}, \text{wavelet.upper_bound}]$ (with default range $[-8, 8]$). $\text{scale} = 1$ corresponds to the case where the extent of the wavelet is $\text{(wavelet.upper_bound - wavelet.lower_bound + 1)}$ samples of the digital signal being analyzed. Larger scales correspond to stretching of the wavelet. For example, at $\text{scale}=10$ the wavelet is stretched by a factor of 10, making it sensitive to lower frequencies in the signal.

To relate a given scale to a specific signal frequency, the sampling period of the signal must be known. `pywt.scale2frequency()` can be used to convert a list of scales to their corresponding frequencies. The proper choice of scales depends on the chosen wavelet, so `pywt.scale2frequency()` should be used to get an idea of an appropriate range for the signal of interest.

For the `cmor`, `fbsp` and `shan` wavelets, the user can specify a specific a normalized center frequency. A value of 1.0 corresponds to $1/\text{dt}$ where $\text{dt}$ is the sampling period. In other words, when analyzing a signal sampled at 100 Hz, a center frequency of 1.0 corresponds to ~100 Hz at $\text{scale} = 1$. This is above the Nyquist rate of 50 Hz, so for this particular wavelet, one would analyze a signal using scales $\geq 2$.

```python
>>> import numpy as np
>>> import pywt
>>> dt = 0.01  # 100 Hz sampling
>>> frequencies = pywt.scale2frequency('cmor1.5-1.0', [1, 2, 3, 4]) / dt
>>> frequencies
array([ 100. ,  50. , 33.33333333,  25. ])
```

The CWT in PyWavelets is applied to discrete data by convolution with samples of the integral of the wavelet. If $\text{scale}$ is too low, this will result in a discrete filter that is inadequately sampled leading to aliasing as shown in the example below. Here the wavelet is 'cmor1.5-1.0'. The left column of the figure shows the discrete filters used in the convolution at various scales. The right column are the corresponding Fourier power spectra of each filter.. For scales 1 and 2 it can be seen that aliasing due to violation of the Nyquist limit occurs.

```python
import numpy as np
import pywt
import matplotlib.pyplot as plt
wav = pywt.ContinuousWavelet('cmor1.5-1.0')
# print the range over which the wavelet will be evaluated
print("Continuous wavelet will be evaluated over the range [{}, {}].".format(wav.lower_bound, wav.upper_bound))
width = wav.upper_bound - wav.lower_bound
scales = [1, 2, 3, 4, 10, 15]
max_len = int(np.max(scales) * width + 1)
t = np.arange(max_len)
fig, axes = plt.subplots(len(scales), 2, figsize=(12, 6))
for n, scale in enumerate(scales):
    # The following code is adapted from the internals of cwt
    int_psi, x = pywt.integrate_wavelet(wav, precision=10)
    step = x[1] - x[0]
    j = np.floor(np.arange(scale * width + 1) / (scale * step))
    if np.max(j) >= np.size(int_psi):
        (continues on next page)
```
j = np.delete(j, np.where((j >= np.size(int_psi)))[0])
j = j.astype(np.int_)

# normalize int_psi for easier plotting
int_psi /= np.abs(int_psi).max()

# discrete samples of the integrated wavelet
filt = int_psi[j][::-1]

# The CWT consists of convolution of filt with the signal at this scale
# Here we plot this discrete convolution kernel at each scale.
nt = len(filt)
t = np.linspace(-nt//2, nt//2, nt)
axes[0, 0].plot(t, filt.real, t, filt.imag)
axes[0, 0].set_xlim([-max_len//2, max_len//2])
axes[0, 0].set_ylim([-1, 1])
axes[0, 0].text(50, 0.35, 'scale = {}

f = np.linspace(-np.pi, np.pi, max_len)
filt_fft = np.fft.fftshift(np.fft.fft(filt, n=max_len))
filt_fft /= np.abs(filt_fft).max()
axes[0, 1].plot(f, np.abs(filt_fft)**2)
axes[0, 1].set_xlim([-np.pi, np.pi])
axes[0, 1].set_ylim([0, 1])
axes[0, 1].set_xticks([-np.pi, 0, np.pi])
axes[0, 1].set_xticklabels([r'$\pi$', '0', r'$\pi$'])
axes[0, 1].grid(True, axis='x')
axes[0, 1].text(np.pi/2, 0.5, 'scale = {}

5.2.14 Thresholding functions

The thresholding helper module implements the most popular signal thresholding functions.

**Thresholding**

```python
pywt.threshold(data, value, mode='soft', substitute=0)
```

Thresholds the input data depending on the mode argument.

In *soft* thresholding [1], data values with absolute value less than `param` are replaced with `substitute`. Data values with absolute value greater or equal to the thresholding value are shrunk toward zero by `value`. In other words, the new value is `data/np.abs(data) * np.maximum(np.abs(data) - value, 0)`.

In *hard* thresholding, the data values where their absolute value is less than the value `param` are replaced with `substitute`. Data values with absolute value greater or equal to the thresholding value stay untouched.

garrote corresponds to the Non-negative garrote threshold [2], [3]. It is intermediate between *hard* and *soft* thresholding. It behaves like soft thresholding for small data values and approaches hard thresholding for
large data values.

In greater thresholding, the data is replaced with substitute where data is below the thresholding value. Greater data values pass untouched.

In less thresholding, the data is replaced with substitute where data is above the thresholding value. Lesser data values pass untouched.

Both hard and soft thresholding also support complex-valued data.

**Parameters**

- **data** [array_like] Numeric data.
- **value** [scalar] Thresholding value.
- **mode** [['soft', 'hard', 'garrote', 'greater', 'less']] Decides the type of thresholding to be applied on input data. Default is 'soft'.
- **substitute** [float, optional] Substitute value (default: 0).

**Returns**

- **output** [array] Thresholded array.

**See also:**

*threshold_firm*

**References**

[1], [2], [3]
Examples

```python
>>> import numpy as np
>>> import pywt
>>> data = np.linspace(1, 4, 7)
>>> data
array([ 1. , 1.5, 2. , 2.5, 3. , 3.5, 4. ])
>>> pywt.threshold(data, 2, 'soft')
array([ 0. , 0. , 0. , 0.5, 1. , 1.5, 2. ])
>>> pywt.threshold(data, 2, 'hard')
array([ 0. , 0. , 2. , 2.5, 3. , 3.5, 4. ])
>>> pywt.threshold(data, 2, 'garrote')
array([ 0. , 0. , 0. , 0.9 , 1.66666667, 2.35714286, 3. ])
>>> pywt.threshold(data, 2, 'greater')
array([ 0. , 0. , 2. , 2.5, 3. , 3.5, 4. ])
>>> pywt.threshold(data, 2, 'less')
array([ 1. , 1.5, 2. , 0. , 0. , 0. , 0. ])
```

```
pywt.threshold_firm(data, value_low, value_high)
Firm threshold.

The approach is intermediate between soft and hard thresholding [1]. It behaves the same as soft-thresholding for values below `value_low` and the same as hard-thresholding for values above `thresh_high`. For intermediate values, the thresholded value is in between that corresponding to soft or hard thresholding.

Parameters

- **data** [array-like] The data to threshold. This can be either real or complex-valued.
- **value_low** [float] Any values smaller then `value_low` will be set to zero.
- **value_high** [float] Any values larger than `value_high` will not be modified.

Returns

- **val_new** [array-like] The values after firm thresholding at the specified thresholds.

See also:

threshold

Notes

This thresholding technique is also known as semi-soft thresholding [2].

For each value, \( x \), in `data`. This function computes:

```python
if np.abs(x) <= value_low:
    return 0
elif np.abs(x) > value_high:
    return x
elif value_low < np.abs(x) and np.abs(x) <= value_high:
    return x * value_high * (1 - value_low/x)/(value_high - value_low)
```

`firm` is a continuous function (like soft thresholding), but is unbiased for large values (like hard thresholding). If `value_high` == `value_low` this function becomes hard-thresholding. If `value_high` is infinity, this function becomes soft-thresholding.
The left panel of the figure below illustrates that non-negative Garotte thresholding is intermediate between soft and hard thresholding. Firm thresholding transitions between soft and hard thresholding behavior. It requires a pair of threshold values that define the width of the transition region.

```python
import numpy as np
import matplotlib.pyplot as plt
import pywt

s = np.linspace(-4, 4, 1000)

s_soft = pywt.threshold(s, value=0.5, mode='soft')
s_hard = pywt.threshold(s, value=0.5, mode='hard')
s_garrote = pywt.threshold(s, value=0.5, mode='garrote')
s_firm1 = pywt.threshold_firm(s, value_low=0.5, value_high=1)
s_firm2 = pywt.threshold_firm(s, value_low=0.5, value_high=2)
s_firm3 = pywt.threshold_firm(s, value_low=0.5, value_high=4)

fig, ax = plt.subplots(1, 2, figsize=(10, 4))
ax[0].plot(s, s_soft)
ax[0].plot(s, s_hard)
ax[0].plot(s, s_garrote)
ax[0].legend(['soft (0.5)', 'hard (0.5)', 'non-neg. garrote (0.5)'])
ax[0].set_xlabel('input value')
ax[0].set_ylabel('thresholded value')

ax[1].plot(s, s_soft)
ax[1].plot(s, s_hard)
ax[1].plot(s, s_firm1)
ax[1].plot(s, s_firm2)
ax[1].plot(s, s_firm3)
ax[1].legend(['soft (0.5)', 'hard (0.5)', 'firm(0.5, 1)', 'firm(0.5, 2)', 'firm(0.5, 4)'])
ax[1].set_xlabel('input value')
ax[1].set_ylabel('thresholded value')
plt.show()
```
5.2.15 Other functions

Integrating wavelet functions

```python
pywt.integrate_wavelet(wavelet, precision=8)
```
Integrate \( \psi \) wavelet function from \(-\infty\) to \(x\) using the rectangle integration method.

**Parameters**

- `wavelet` [Wavelet instance or str] Wavelet to integrate. If a string, should be the name of a wavelet.
- `precision` [int, optional] Precision that will be used for wavelet function approximation computed with the `wavefun(level=precision)` Wavelet’s method (default: 8).

**Returns**

- `[int_psi, x]` : for orthogonal wavelets
- `[int_psi_d, int_psi_r, x]` : for other wavelets

**Examples**

```python
>>> from pywt import Wavelet, integrate_wavelet
>>> wavelet1 = Wavelet('db2')
>>> [int_psi, x] = integrate_wavelet(wavelet1, precision=5)
>>> wavelet2 = Wavelet('bior1.3')
>>> [int_psi_d, int_psi_r, x] = integrate_wavelet(wavelet2, precision=5)
```

The result of the call depends on the `wavelet` argument:

- for orthogonal and continuous wavelets - an integral of the wavelet function specified on an `x`-grid:
  ```python
  [int_psi, x_grid] = integrate_wavelet(wavelet, precision)
  ```
- for other wavelets - integrals of decomposition and reconstruction wavelet functions and a corresponding `x`-grid:
  ```python
  [int_psi_d, int_psi_r, x_grid] = integrate_wavelet(wavelet, precision)
  ```

Central frequency of \( \psi \) wavelet function

```python
pywt.central_frequency(wavelet, precision=8)
```
Computes the central frequency of the \( \psi \) wavelet function.

**Parameters**

- `wavelet` [Wavelet instance, str or tuple] Wavelet to integrate. If a string, should be the name of a wavelet.
- `precision` [int, optional] Precision that will be used for wavelet function approximation computed with the `wavefun(level=precision)` Wavelet’s method (default: 8).

**Returns**

- `scalar`

```python
pywt.scale2frequency(wavelet, scale, precision=8)
```
**Parameters**
wavelet [Wavelet instance or str] Wavelet to integrate. If a string, should be the name of a wavelet.

scale [scalar]

precision [int, optional] Precision that will be used for wavelet function approximation computed with \texttt{wavelet.wavefun(level=precision)}. Default is 8.

Returns

freq [scalar]

Quadrature Mirror Filter

\texttt{pywt.qmf(filt)}

Returns the Quadrature Mirror Filter (QMF).

The magnitude response of QMF is mirror image about $\pi/2$ of that of the input filter.

Parameters

filt [array_like] Input filter for which QMF needs to be computed.

Returns

qm_filter [ndarray] Quadrature mirror of the input filter.

Orthogonal Filter Banks

\texttt{pywt.orthogonal_filter_bank(scaling_filter)}

Returns the orthogonal filter bank.

The orthogonal filter bank consists of the HPFs and LPFs at decomposition and reconstruction stage for the input scaling filter.

Parameters

scaling_filter [array_like] Input scaling filter (father wavelet).

Returns


Example Datasets

The following example datasets are available in the module pywt.data:

<table>
<thead>
<tr>
<th>name</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ecg</td>
<td>ECG waveform (1024 samples)</td>
</tr>
<tr>
<td>aero</td>
<td>grayscale image (512x512)</td>
</tr>
<tr>
<td>ascent</td>
<td>grayscale image (512x512)</td>
</tr>
<tr>
<td>camera</td>
<td>grayscale image (512x512)</td>
</tr>
<tr>
<td>nino</td>
<td>sea surface temperature (264 samples)</td>
</tr>
<tr>
<td>demo_signal</td>
<td>various synthetic 1d test signals</td>
</tr>
</tbody>
</table>

Each can be loaded via a function of the same name.
pywt.data.demo_signal (name='Bumps', n=None)
Simple 1D wavelet test functions.

This function can generate a number of common 1D test signals used in papers by David Donoho and colleagues (e.g. [1]) as well as the wavelet book by Stéphane Mallat [2].

**Parameters**

- **name** [‘Blocks’, ‘Bumps’, ‘HeaviSine’, ‘Doppler’, …] The type of test signal to generate (name is case-insensitive). If name is set to ‘list’, a list of the available test functions is returned.

- **n** [int or None] The length of the test signal. This should be provided for all test signals except ‘Gabor’ and ‘sineoneoverx’ which have a fixed length.

**Returns**

- **f** [np.ndarray] Array of length n corresponding to the specified test signal type.

**Notes**

This function is a partial reimplementation of the MakeSignal function from the [Wavelab](https://statweb.stanford.edu/~wavelab/) toolbox. These test signals are provided with permission of Dr. Donoho to encourage reproducible research.

**References**

[1], [2]

**Example:**

```python
>>> import pywt
>>> camera = pywt.data.camera()
>>> doppler = pywt.data.demo_signal('doppler', 1024)
>>> available_signals = pywt.data.demo_signal('list')
```

5.3 Usage examples

The following examples are used as doctest regression tests written using reST markup. They are included in the documentation since they contain various useful examples illustrating how to use and how not to use PyWavelets.

For more usage examples see the demo directory in the source package.

5.3.1 The Wavelet object

**Wavelet families and builtin Wavelets names**

Wavelet objects are really a handy carriers of a bunch of DWT-specific data like quadrature mirror filters and some general properties associated with them.

At first let’s go through the methods of creating a Wavelet object. The easiest and the most convenient way is to use builtin named Wavelets.

These wavelets are organized into groups called wavelet families. The most commonly used families are:
>>> import pywt
>>> pywt.families()
['haar', 'db', 'sym', 'coif', 'bior', 'rbio', 'dmey', 'gaus', 'mexh', 'morl', 'cgau', 'shan', 'fbsp', 'cmor']

The `wavelist()` function with family name passed as an argument is used to obtain the list of wavelet names in each family.

```python
>>> for family in pywt.families():
    ...   print(f"{family} family: ", ', '.join(pywt.wavelist(family)))
haar family: haar
db family: db1, db2, db3, db4, db5, db6, db7, db8, db9, db10, db11, db12, db13, db14,
    db15, db16, db17, db18, db19, db20, db21, db22, db23, db24, db25, db26, db27, db28,
    db29, db30, db31, db32, db33, db34, db35, db36, db37, db38
sym family: sym2, sym3, sym4, sym5, sym6, sym7, sym8, sym9, sym10, sym11, sym12,
    sym13, sym14, sym15, sym16, sym17, sym18, sym19, sym20
coif family: coif1, coif2, coif3, coif4, coif5, coif6, coif7, coif8, coif9, coif10,
    coif11, coif12, coif13, coif14, coif15, coif16, coif17
bior family: bior1.1, bior1.3, bior1.5, bior1.7, bior1.9, bior2.2, bior2.4, bior2.6, bior2.8, bior3.1,
    bior3.3, bior3.5, bior3.7, bior3.9, bior4.4, bior5.5, bior6.8
rbio family: rbio1.1, rbio1.3, rbio1.5, rbio1.7, rbio1.9, rbio2.2, rbio2.4, rbio2.6, rbio2.8, rbio3.1,
    rbio3.3, rbio3.5, rbio3.7, rbio3.9, rbio4.4, rbio5.5, rbio6.8
dmey family: dmey
gaus family: gaus1, gaus2, gaus3, gaus4, gaus5, gaus6, gaus7, gaus8
mexh family: mexh
morl family: morl
cgau family: cgau1, cgau2, cgau3, cgau4, cgau5, cgau6, cgau7, cgau8
shan family: shan
fbsp family: fbsp
cmor family: cmor
```

To get the full list of builtin wavelets’ names just use the `wavelist()` with no argument.

**Creating Wavelet objects**

Now when we know all the names let’s finally create a `Wavelet` object:

```python
>>> w = pywt.Wavelet('db3')
```

So.. that’s it.

**Wavelet properties**

But what can we do with `Wavelet` objects? Well, they carry some interesting information.

First, let’s try printing a `Wavelet` object. This shows a brief information about its name, its family name and some properties like orthogonality and symmetry.

```python
>>> print(w)
Wavelet db3
  Family name: Daubechies
  Short name: db
  Filters length: 6
  Orthogonal: True
  Biorthogonal: True
```

(continues on next page)
But the most important information are the wavelet filters coefficients, which are used in Discrete Wavelet Transform. These coefficients can be obtained via the `dec_lo`, `Wavelet.dec_hi`, `rec_lo` and `rec_hi` attributes, which corresponds to lowpass and highpass decomposition filters and lowpass and highpass reconstruction filters respectively:

```python
>>> def print_array(arr):
...     print("[", ", ".join(["%.14f" % x for x in arr])
```

Another way to get the filters data is to use the `filter_bank` attribute, which returns all four filters in a tuple:

```python
>>> w.filter_bank == (w.dec_lo, w.dec_hi, w.rec_lo, w.rec_hi)
True
```

Other Wavelet's properties are:

Wavelet `name`, `short_family_name` and `family_name`:

```python
>>> print(w.name)
db3
>>> print(w.short_family_name)
db
>>> print(w.family_name)
Daubechies
```

- Decomposition (`dec_len`) and reconstruction (`rec_len`) filter lengths:

```python
>>> int(w.dec_len)  # int() is for normalizing longs and ints for doctest
6
>>> int(w.rec_len)
6
```

- Orthogonality (`orthogonal`) and biorthogonality (`biorthogonal`):

```python
>>> w.orthogonal
True
>>> w.biorthogonal
True
```

- Symmetry (`symmetry`):

```python
>>> print(w.symmetry)
asymmetric
```

- Number of vanishing moments for the scaling function phi (`vanishing_moments_phi`) and the wavelet function psi (`vanishing_moments_psi`) associated with the filters:

```python
>>> w.vanishing_moments_phi
0
>>> w.vanishing_moments_psi
3
```

Now when we know a bit about the builtin Wavelets, let’s see how to create custom Wavelets objects. These can be done in two ways:
1) Passing the filter bank object that implements the `filter_bank` attribute. The attribute must return four filters coefficients.

```python
>>> class MyHaarFilterBank(object):
...     @property
...     def filter_bank(self):
...         from math import sqrt
...         return ([sqrt(2)/2, sqrt(2)/2], [-sqrt(2)/2, sqrt(2)/2],
...                 [sqrt(2)/2, sqrt(2)/2], [sqrt(2)/2, -sqrt(2)/2])

>>> my_wavelet = pywt.Wavelet('My Haar Wavelet', filter_bank=MyHaarFilterBank())
```

2) Passing the filters coefficients directly as the `filter_bank` parameter.

```python
>>> from math import sqrt
>>> my_filter_bank = ([sqrt(2)/2, sqrt(2)/2], [-sqrt(2)/2, sqrt(2)/2],
...                    [sqrt(2)/2, sqrt(2)/2], [sqrt(2)/2, -sqrt(2)/2])
>>> my_wavelet = pywt.Wavelet('My Haar Wavelet', filter_bank=my_filter_bank)
```

Note that such custom wavelets will not have all the properties set to correct values:

```python
>>> print(my_wavelet)
Wavelet My Haar Wavelet
  Family name: 
  Short name: 
  Filters length: 2
  Orthogonal: False
  Biorthogonal: False
  Symmetry: unknown
  DWT: True
  CWT: False
```

You can however set a couple of them on your own:

```python
>>> my_wavelet.orthogonal = True
>>> my_wavelet.biorthogonal = True
```

```python
>>> print(my_wavelet)
Wavelet My Haar Wavelet
  Family name: 
  Short name: 
  Filters length: 2
  Orthogonal: True
  Biorthogonal: True
  Symmetry: unknown
  DWT: True
  CWT: False
```

And now... the wavefun!

We all know that the fun with wavelets is in wavelet functions. Now what would be this package without a tool to compute wavelet and scaling functions approximations?

This is the purpose of the `wavefun()` method, which is used to approximate scaling function (\(\phi\)) and wavelet function (\(\psi\)) at the given level of refinement, based on the filters coefficients.

5.3. Usage examples
The number of returned values varies depending on the wavelet’s orthogonality property. For orthogonal wavelets the result is tuple with scaling function, wavelet function and xgrid coordinates.

```python
>>> w = pywt.Wavelet('sym3')
>>> w.orthogonal
True
>>> (phi, psi, x) = w.wavefun(level=5)
```

For biorthogonal (non-orthogonal) wavelets different scaling and wavelet functions are used for decomposition and reconstruction, and thus five elements are returned: decomposition scaling and wavelet functions approximations, reconstruction scaling and wavelet functions approximations, and the xgrid.

```python
>>> w = pywt.Wavelet('bior1.3')
>>> w.orthogonal
False
>>> (phi_d, psi_d, phi_r, psi_r, x) = w.wavefun(level=5)
```

See also:
You can find live examples of `wavefun()` usage and images of all the built-in wavelets on the Wavelet Properties Browser page. However, this website is no longer actively maintained and does not include every wavelet present in PyWavelets. The precision of the wavelet coefficients at that site is also lower than those included in PyWavelets.

### 5.3.2 Signal Extension Modes

Import `pywt` first

```python
>>> import pywt
```

```python
>>> def format_array(a):
...     """Consistent array representation across different systems"
...     import numpy
...     a = numpy.where(numpy.abs(a) < 1e-5, 0, a)
...     return numpy.array2string(a, precision=5, separator=' ', suppress_small=True)
```

List of available signal extension *modes*:

```python
>>> print(pywt.Modes.modes)
['zero', 'constant', 'symmetric', 'periodic', 'smooth', 'periodization', 'reflect', 'antisymmetric', 'antireflect']
```

Invalid mode name should rise a `ValueError`:

```python
>>> pywt.dwt([1,2,3,4], 'db2', 'invalid')
Traceback (most recent call last):
...
ValueError: Unknown mode name 'invalid'.
```

You can also refer to modes via `Modes` class attributes:

```python
>>> x = [1, 2, 1, 5, -1, 8, 4, 6]
>>> for mode_name in ['zero', 'constant', 'symmetric', 'reflect', 'periodic', 'smooth', 'periodization']:
...     mode = getattr(pywt.Modes, mode_name)
...     cA, cD = pywt.dwt(x, 'db2', mode)
...     print("Mode: %d (%s)" % (mode, mode_name))
```
Mode: 0 (zero)
Mode: 2 (constant)
Mode: 1 (symmetric)
Mode: 6 (reflect)
Mode: 4 (periodic)
Mode: 3 (smooth)
Mode: 5 (periodization)

The default mode is symmetric:

```python
>>> cA, cD = pywt.dwt(x, 'db2')
>>> print(cA)
[ 1.76776695 1.73309178 3.40612438 6.32928585 7.77817459]
>>> print(cD)
[-0.61237244 -2.15599552 -5.95034847 -1.21545369 1.22474487]
>>> print(pywt.idwt(cA, cD, 'db2'))
[ 1. 2. 1. 5. -1. 8. 4. 6.]
```

And using a keyword argument:

```python
>>> cA, cD = pywt.dwt(x, 'db2', mode='symmetric')
>>> print(cA)
[ 1.76776695 1.73309178 3.40612438 6.32928585 7.77817459]
>>> print(cD)
[-0.61237244 -2.15599552 -5.95034847 -1.21545369 1.22474487]
>>> print(pywt.idwt(cA, cD, 'db2'))
[ 1. 2. 1. 5. -1. 8. 4. 6.]
```

### 5.3.3 DWT and IDWT

#### Discrete Wavelet Transform

Let's do a Discrete Wavelet Transform of a sample data $x$ using the db2 wavelet. It's simple..

```python
>>> import pywt

>>> x = [3, 7, 1, 1, -2, 5, 4, 6]
>>> cA, cD = pywt.dwt(x, 'db2')
```

And the approximation and details coefficients are in $cA$ and $cD$ respectively:

```python
>>> print(cA)
[ 5.65685425 7.39923721 0.22414387 3.33677403 7.77817459]
>>> print(cD)
[-2.44948974 -1.60368225 -4.44140056 -0.41361256 1.22474487]
```

#### Inverse Discrete Wavelet Transform

Now let's do an opposite operation - Inverse Discrete Wavelet Transform:

```python
>>> print(pywt.idwt(cA, cD, 'db2'))
[ 3. 7. 1. 1. -2. 5. 4. 6.]
```

Voilà! That's it!
More Examples

Now let’s experiment with the \texttt{dwt()} some more. For example let’s pass a \texttt{Wavelet} object instead of the wavelet name and specify signal extension mode (the default is \texttt{symmetric}) for the border effect handling:

```
>>> w = pywt.Wavelet('sym3')
>>> cA, cD = pywt.dwt(x, wavelet=w, mode='constant')
>>> print(cA)
[ 4.38354585  3.80302657  7.31813271 -0.58565539  4.09727044  7.81994027]
>>> print(cD)
[-1.33068221 -2.78795192 -3.16825651 -0.67715519 -0.09722957 -0.07045258]
```

Note that the output coefficients arrays length depends not only on the input data length but also on the \texttt{Wavelet} type (particularly on its \texttt{filters} length that are used in the transformation).

To find out what will be the output data size use the \texttt{dwt_coeff_len()} function:

```
>>> # int() is for normalizing Python integers and long integers for documentation
... tests
>>> int(pywt.dwt_coeff_len(data_len=len(x), filter_len=w.dec_len, mode='symmetric'))
6
>>> int(pywt.dwt_coeff_len(len(x), w, 'symmetric'))
6
>>> len(cA)
6
```

Looks fine. (And if you expected that the output length would be a half of the input data length, well, that’s the trade-off that allows for the perfect reconstruction...).

The third argument of the \texttt{dwt_coeff_len()} is the already mentioned signal extension mode (please refer to the PyWavelets’ documentation for the \texttt{modes} description). Currently there are six extension modes available:

```
>>> pywt.Modes.modes
['zero', 'constant', 'symmetric', 'periodic', 'smooth', 'periodization', 'reflect',
 'antisymmetric', 'antireflect']
```

As you see in the above example, the \texttt{periodization} (periodization) mode is slightly different from the others. It’s aim when doing the \texttt{DWT} transform is to output coefficients arrays that are half of the length of the input data.

Knowing that, you should never mix the periodization mode with other modes when doing \texttt{DWT} and \texttt{IDWT}. Otherwise, it will produce invalid results:

```
>>> x
[3, 7, 1, 1, -2, 5, 4, 6]
>>> cA, cD = pywt.dwt(x, wavelet=w, mode='periodization')
>>> print(pywt.idwt(cA, cD, 'sym3', 'symmetric')) # invalid mode
[ 1.  1. -2.  5.]
>>> print(pywt.idwt(cA, cD, 'sym3', 'periodization'))
[ 3.  7.  1.  1. -2.  5.  4.  6.]
```

Tips & tricks

\textbf{Passing None instead of coefficients data to \texttt{idwt()}}

Now some tips & tricks. Passing \texttt{None} as one of the coefficient arrays parameters is similar to passing a zero-filled array. The results are simply the same:
>>> print(pywt.idwt([1, 2, 0, 1], None, 'db2', 'symmetric'))
[ 1.19006969  1.54362308  0.44828774  0.25881905  0.48296291  0.8365163 ]

>>> print(pywt.idwt([1, 2, 0, 1], [0, 0, 0, 0], 'db2', 'symmetric'))
[ 1.19006969  1.54362308  0.44828774  0.25881905  0.48296291  0.8365163 ]

>>> print(pywt.idwt(None, [1, 2, 0, 1], 'db2', 'symmetric'))
[ 0.57769726 -0.93125065  1.67303261 -0.96592583 -0.12940952 -0.22414387]

>>> print(pywt.idwt([0, 0, 0, 0], [1, 2, 0, 1], 'db2', 'symmetric'))
[ 0.57769726 -0.93125065  1.67303261 -0.96592583 -0.12940952 -0.22414387]

Remember that only one argument at a time can be None:

>>> print(pywt.idwt(None, None, 'db2', 'symmetric'))
Traceback (most recent call last):
...
ValueError: At least one coefficient parameter must be specified.

Coefficients data size in idwt

When doing the IDWT transform, usually the coefficient arrays must have the same size.

>>> print(pywt.idwt([1, 2, 3, 4, 5], [1, 2, 3, 4], 'db2', 'symmetric'))
Traceback (most recent call last):
...
ValueError: Coefficients arrays must have the same size.

Not every coefficient array can be used in IDWT. In the following example the idwt() will fail because the input arrays are invalid - they couldn’t be created as a result of DWT, because the minimal output length for dwt using db4 wavelet and the symmetric mode is 4, not 3:

>>> pywt.idwt([1, 2, 4], [4, 1, 3], 'db4', 'symmetric')
Traceback (most recent call last):
...
ValueError: Invalid coefficient arrays length for specified wavelet. Wavelet and mode must be the same as used for decomposition.

>>> int(pywt.dwt_coeff_len(1, pywt.Wavelet('db4').dec_len, 'symmetric'))
4

5.3.4 Multilevel DWT, IDWT and SWT

Multilevel DWT decomposition

>>> import pywt
>>> x = [3, 7, 1, 1, -2, 5, 4, 6]
>>> db1 = pywt.Wavelet('db1')
>>> ca3, cd3, cd2, cd1 = pywt.wavedec(x, db1)
>>> print(ca3)
[ 8.83883476]
>>> print(cD3)
[-0.35355339]
>>> print(cD2)
[ 4. -3.5]
>>> print(cD1)
[-2.82842712  0.  -4.94974747 -1.41421356]

>>> pywt.dwt_max_level(len(x), db1)
3

>>> cA2, cD2, cD1 = pywt.wavedec(x, db1, mode='constant', level=2)

Multilevel IDWT reconstruction

>>> coeffs = pywt.wavedec(x, db1)
>>> print(pywt.waverec(coeffs, db1))
[ 3.  7.  1.  1. -2.  5.  4.  6.]

Multilevel SWT decomposition

>>> x = [3, 7, 1, 3, -2, 6, 4, 6]
>>> (cA2, cD2), (cA1, cD1) = pywt.swt(x, db1, level=2)

>>> print(cA1)
[ 7.07106781  5.65685425  2.82842712  0.70710678  2.82842712  7.07106781  7.07106781  6.36396103]

>>> print(cD1)
[-2.82842712  4.24264069 -1.41421356  3.53553391 -5.65685425  1.41421356  -1.41421356  2.12132034]

>>> print(cA2)
[ 7.  4.5  4.  5.5  7.  9.5 10.  8.5]

>>> print(cD2)
[ 3.  3.5  0. -4.5 -3.  0.5  0.  0.5]

>>> [(cA2, cD2)] = pywt.swt(cA1, db1, level=1, start_level=1)

>>> print(cA2)
[ 7.  4.5  4.  5.5  7.  9.5 10.  8.5]

>>> print(cD2)
[ 3.  3.5  0. -4.5 -3.  0.5  0.  0.5]

>>> coeffs = pywt.swt(x, db1)
>>> len(coeffs)
3
>>> pywt.swt_max_level(len(x))
3

>>> from __future__ import print_function

5.3.5 Wavelet Packets
**Import pywt**

```python
>>> import pywt

>>> def format_array(a):
...     """Consistent array representation across different systems"""
...     import numpy
...     a = numpy.where(numpy.abs(a) < 1e-5, 0, a)
...     return numpy.array2string(a, precision=5, separator=' ', suppress_small=True)
```

**Create Wavelet Packet structure**

Ok, let’s create a sample `WaveletPacket`:

```python
>>> x = [1, 2, 3, 4, 5, 6, 7, 8]
>>> wp = pywt.WaveletPacket(data=x, wavelet='db1', mode='symmetric')
```

The input `data` and decomposition coefficients are stored in the `WaveletPacket.data` attribute:

```python
>>> print(wp.data)
[1, 2, 3, 4, 5, 6, 7, 8]
```

Nodes are identified by paths. For the root node the path is `'` and the decomposition level is 0.

```python
>>> print(repr(wp.path))
'

>>> print(wp.level)
0
```

The `maxlevel`, if not given as param in the constructor, is automatically computed:

```python
>>> print(wp['ad'].maxlevel)
3
```

**Traversing WP tree:**

**Accessing subnodes:**

```python
>>> x = [1, 2, 3, 4, 5, 6, 7, 8]
>>> wp = pywt.WaveletPacket(data=x, wavelet='db1', mode='symmetric')
```

First check what is the maximum level of decomposition:

```python
>>> print(wp.maxlevel)
3
```

and try accessing subnodes of the WP tree:

- 1st level:

```python
>>> print(wp['a'].data)
[ 2.12132034 4.94974747 7.77817459 10.60660172]
```

---

**5.3. Usage examples**
• 2nd level:

```python
>>> print(wp['aa'].data)
[  5.  13.]
>>> print(wp['aa'].path)
aa
```

• 3rd level:

```python
>>> print(wp['aaa'].data)
[ 12.72792206]
>>> print(wp['aaa'].path)
aaa
```

Ups, we have reached the maximum level of decomposition and got an `IndexError`:

```python
>>> print(wp['aaaa'].data)
Traceback (most recent call last):
...  
IndexError: Path length is out of range.
```

Now try some invalid path:

```python
>>> print(wp['ac'])
Traceback (most recent call last):
...  
ValueError: Subnode name must be in ['a', 'd'], not 'c'.
```

which just yielded a `ValueError`.

**Accessing Node’s attributes:**

`WaveletPacket` object is a tree data structure, which evaluates to a set of `Node` objects. `WaveletPacket` is just a special subclass of the `Node` class (which in turn inherits from the `BaseNode`).

Tree nodes can be accessed using the `obj[x]` (`Node.__getitem__()`) operator. Each tree node has a set of attributes: `data`, `path`, `node_name`, `parent`, `level`, `maxlevel` and `mode`.

```python
>>> x = [1, 2, 3, 4, 5, 6, 7, 8]
>>> wp = pywt.WaveletPacket(data=x, wavelet='db1', mode='symmetric')

>>> print(wp['ad'].data)
[[-2. -2.]]

>>> print(wp['ad'].path)
ad

>>> print(wp['ad'].node_name)
d
>>> print(wp['ad'].parent.path)
a

>>> print(wp['ad'].level)
2
```
Collecting nodes

```python
>>> x = [1, 2, 3, 4, 5, 6, 7, 8]
>>> wp = pywt.WaveletPacket(data=x, wavelet='db1', mode='symmetric')

We can get all nodes on the particular level either in natural order:

```python
>>> print(['aaa', 'aad', 'ada', 'add', 'daa', 'dad', 'dda', 'ddd'])
```
Removing nodes from Wavelet Packet tree:

Let’s create a sample data:

```python
>>> x = [1, 2, 3, 4, 5, 6, 7, 8]
>>> wp = pywt.WaveletPacket(data=x, wavelet='db1', mode='symmetric')
```

First, start with a tree decomposition at level 2. Leaf nodes in the tree are:

```python
>>> dummy = wp.get_level(2)
>>> for n in wp.get_leaf_nodes(False):
...     print(n.path, format_array(n.data))
'aa' [ 5. 13.]
'ad' [-2. -2.]
'da' [-1. -1.]
'dd' [ 0. 0.]
```

```python
>>> node = wp['ad']
>>> print(node)
ad: [-2. -2.]
```

To remove a node from the WP tree, use Python’s `del obj[x]` (Node.__delitem__):

```python
>>> del wp['ad']
```

The leaf nodes that left in the tree are:

```python
>>> for n in wp.get_leaf_nodes():
...     print(n.path, format_array(n.data))
'aa' [ 5. 13.]
'da' [-1. -1.]
'dd' [ 0. 0.]
```

And the reconstruction is:

```python
>>> print(wp.reconstruct())
[ 2. 3. 2. 3. 6. 7. 6. 7.]
```

Now restore the deleted node value.

```python
>>> wp['ad'].data = node.data
```

Printing leaf nodes and tree reconstruction confirms the original state of the tree:
>>> for n in wp.get_leaf_nodes(False):
...     print(n.path, format_array(n.data))
aa [ 5. 13.]
ad [-2. -2.]
da [-1. -1.]
dd [ 0. 0.]

>>> print(wp.reconstruct())
[ 1. 2. 3. 4. 5. 6. 7. 8.]

Lazy evaluation:

Note: This section is for demonstration of pywt internals purposes only. Do not rely on the attribute access to nodes as presented in this example.

>>> x = [1, 2, 3, 4, 5, 6, 7, 8]
>>> wp = pywt.WaveletPacket(data=x, wavelet='db1', mode='symmetric')

1) At first the wp’s attribute a is None

>>> print(wp.a)
None

Remember that you should not rely on the attribute access.

2) At first attempt to access the node it is computed via decomposition of its parent node (the wp object itself).

>>> print(wp['a'])
a: [ 2.12132034 4.94974747 7.77817459 10.60660172]

3) Now the wp.a is set to the newly created node:

>>> print(wp.a)
a: [ 2.12132034 4.94974747 7.77817459 10.60660172]

And so is wp.d:

>>> print(wp.d)
d: [-0.70710678 -0.70710678 -0.70710678 -0.70710678]

5.3.6 2D Wavelet Packets

Import pywt

>>> from __future__ import print_function
>>> import pywt
>>> import numpy

5.3. Usage examples
Create 2D Wavelet Packet structure

Start with preparing test data:

```python
>>> x = numpy.array([[1, 2, 3, 4, 5, 6, 7, 8]] * 8, 'd')
>>> print(x)
[[ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]]
```

Now create a 2D Wavelet Packet object:

```python
>>> wp = pywt.WaveletPacket2D(data=x, wavelet='db1', mode='symmetric')
```

The input data and decomposition coefficients are stored in the `WaveletPacket2D.data` attribute:

```python
>>> print(wp.data)
[[ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]]
```

Nodes are identified by paths. For the root node the path is '' and the decomposition level is 0.

```python
>>> print(repr(wp.path))
''
>>> print(wp.level)
0
```

The `WaveletPacket2D.maxlevel`, if not given in the constructor, is automatically computed based on the data size:

```python
>>> print(wp.maxlevel)
3
```

**Traversing WP tree:**

Wavelet Packet nodes are arranged in a tree. Each node in a WP tree is uniquely identified and addressed by a path string.

In the 1D `WaveletPacket` case nodes were accessed using 'a' (approximation) and 'd' (details) path names (each node has two 1D children).

Because now we deal with a bit more complex structure (each node has four children), we have four basic path names based on the dwt 2D output convention to address the WP2D structure:

- a - LL, low-low coefficients
- h - LH, low-high coefficients
- v - HL, high-low coefficients
- d - HH, high-high coefficients

In other words, subnode naming corresponds to the `dwt2()` function output naming convention (as wavelet packet transform is based on the dwt2 transform):

\[
(cA, (cH, cV, cD)) \leftrightarrow \begin{array}{c}
\text{cA(LL)} \\
\text{cH(LH)} \\
\text{cV(HL)} \\
\text{cD(HH)}
\end{array}
\]

(fig.1: DWT 2D output and interpretation)

Knowing what the nodes names are, we can now access them using the indexing operator `obj[x]` (WaveletPacket2D.__getitem__()):

```python
>>> print(wp['a'].data)
[[ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]]

>>> print(wp['h'].data)
[[ 0.  0.  0.  0.]
 [ 0.  0.  0.  0.]
 [ 0.  0.  0.  0.]
 [ 0.  0.  0.  0.]]

>>> print(wp['v'].data)
[[ -1. -1. -1. -1.]
 [ -1. -1. -1. -1.]
 [ -1. -1. -1. -1.]
 [ -1. -1. -1. -1.]]

>>> print(wp['d'].data)
[[ 0.  0.  0.  0.]
 [ 0.  0.  0.  0.]
 [ 0.  0.  0.  0.]
 [ 0.  0.  0.  0.]]
```

Similarly, a subnode of a subnode can be accessed by:

```python
>>> print(wp['aa'].data)
[[ 10. 26.]
 [ 10. 26.]]
```

Indexing base `WaveletPacket2D` (as well as 1D `WaveletPacket`) using compound path is just the same as indexing WP subnode:

```python
>>> node = wp['a']

>>> print(node['a'].data)
[[ 10. 26.]
 [ 10. 26.]]

>>> print(wp['a']['a'].data is wp['aa'].data)
True
```
Following down the decomposition path:

```python
>>> print(wp['aaa'].data)
[[ 36.]]
>>> print(wp['aaaa'].data)
Traceback (most recent call last):
...  
IndexError: Path length is out of range.
```

Ups, we have reached the maximum level of decomposition for the 'aaaa' path, which btw. was:

```python
>>> print(wp.maxlevel)
3
```

Now try some invalid path:

```python
>>> print(wp['f'])
Traceback (most recent call last):
...  
ValueError: Subnode name must be in ['a', 'h', 'v', 'd'], not 'f'.
```

### Accessing Node2D's attributes:

WaveletPacket2D is a tree data structure, which evaluates to a set of Node2D objects. WaveletPacket2D is just a special subclass of the Node2D class (which in turn inherits from a BaseNode, just like with Node and WaveletPacket for the 1D case.).

```python
>>> print(wp['av'].data)
[[ -4. -4.]
 [ -4. -4.]]
```

```python
>>> print(wp['av'].path)
av
```

```python
>>> print(wp['av'].node_name)
v
```

```python
>>> print(wp['av'].parent.path)
a
```

```python
>>> print(wp['av'].parent.data)
[[  3.  11.  15.]
 [  3.  11.  15.]
 [  3.  11.  15.]]
```

```python
>>> print(wp['av'].level)
2
```

```python
>>> print(wp['av'].maxlevel)
3
```
Collecting nodes

We can get all nodes on the particular level using the `WaveletPacket2D.get_level()` method:

- 0 level - the root `wp` node:

```python
>>> len(wp.get_level(0))
1
>>> print([node.path for node in wp.get_level(0)])
['']
```

- 1st level of decomposition:

```python
>>> len(wp.get_level(1))
4
>>> print([node.path for node in wp.get_level(1)])
['a', 'h', 'v', 'd']
```

- 2nd level of decomposition:

```python
>>> len(wp.get_level(2))
16
>>> paths = [node.path for node in wp.get_level(2)]
>>> for i, path in enumerate(paths):
...     if (i+1) % 4 == 0:
...         print(path)
...     else:
...         print(path, end=' ')
aaa aah av ad
ha hh hv hd
va vh vv vd
da dh dv dd
```

- 3rd level of decomposition:

```python
>>> print(len(wp.get_level(3)))
64
>>> paths = [node.path for node in wp.get_level(3)]
>>> for i, path in enumerate(paths):
...     if (i+1) % 8 == 0:
...         print(path)
...     else:
...         print(path, end=' ')
aaa aah aav aad aha ahh ahv ab hd
ava avh avv ava ada adh adv add
haa hah hav had hha hhh hhv hhb hbd
hva hvb hvv hvd hda hdb hdv hdd
vaa vah vav vad vha vhh vhp vhd
vva vvb vvv vvd vda vdh vdv vdd
da aha hav dad dha dhv dhb dhd
da aha hav dad dha dhv dhb dhd
da aha hav dad dha dhv dhb dhd
da aha hav dad dha dhv dhb dhd
da aha hav dad dha dhv dhb dhd
da aha hav dad dha dhv dhb dhd
da aha hav dad dha dhv dhb dhd
da aha hav dad dha dhv dhb dhd
```

Note that `WaveletPacket2D.get_level()` performs automatic decomposition until it reaches the given level.

5.3. Usage examples
Reconstructing data from Wavelet Packets:

Let's create a new empty 2D Wavelet Packet structure and set its nodes values with known data from the previous examples:

```python
>>> new_wp = pywt.WaveletPacket2D(data=None, wavelet='db1', mode='symmetric')
```

```python
>>> new_wp['vh'] = wp['vh'].data # [[0.0, 0.0], [0.0, 0.0]]
>>> new_wp['vv'] = wp['vh'].data # [[0.0, 0.0], [0.0, 0.0]]
>>> new_wp['vd'] = [[0.0, 0.0], [0.0, 0.0]]
>>> new_wp['a'] = [[3.0, 7.0, 11.0, 15.0], [3.0, 7.0, 11.0, 15.0], ...
     ... [3.0, 7.0, 11.0, 15.0], [3.0, 7.0, 11.0, 15.0]]
>>> new_wp['d'] = [[0.0, 0.0, 0.0, 0.0], [0.0, 0.0, 0.0, 0.0], ...
     ... [0.0, 0.0, 0.0, 0.0], [0.0, 0.0, 0.0, 0.0]]
```

For convenience, `Node2D.data` gets automatically extracted from the base `Node2D` object:

```python
>>> new_wp['h'] = wp['h'] # all zeros
```

Note: just remember to not assign to the node.data parameter directly (todo).

And reconstruct the data from the a, d, vh, vv, vd and h packets (Note that va node was not set and the WP tree is “not complete” - the va branch will be treated as `zero-array`):

```python
>>> print(new_wp.reconstruct(update=False))
[[ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]]
```

Now set the va node with the known values and do the reconstruction again:

```python
>>> new_wp['va'] = wp['va'].data # [[-2.0, -2.0], [-2.0, -2.0]]
```  

```python
>>> print(new_wp.reconstruct(update=False))
[[ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]]
```

which is just the same as the base sample data x.

Of course we can go the other way and remove nodes from the tree. If we delete the va node, again, we get the “not complete” tree from one of the previous examples:

```python
>>> del new_wp['va']
```  

```python
>>> print(new_wp.reconstruct(update=False))
[[ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]
 [ 1.5  1.5  3.5  3.5  5.5  5.5  7.5  7.5]]
```

(continues on next page)
Just restore the node before next examples.

```python
>>> new_wp['va'] = wp['va'].data
```

If the `update` param in the `WaveletPacket2D.reconstruct()` method is set to `False`, the node's `Node2D.data` attribute will not be updated.

```python
>>> print(new_wp.data)
None
```

Otherwise, the `WaveletPacket2D.data` attribute will be set to the reconstructed value.

```python
>>> print(new_wp.reconstruct(update=True))
[[ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]]
```

```python
>>> print(new_wp.data)
[[ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]
 [ 1.  2.  3.  4.  5.  6.  7.  8.]]
```

Since we have an interesting WP structure built, it is a good occasion to present the `WaveletPacket2D.get_leaf_nodes()` method, which collects non-zero leaf nodes from the WP tree:

```python
>>> print([n.path for n in new_wp.get_leaf_nodes()])
['a', 'h', 'va', 'vh', 'vv', 'vd', 'd']
```

Passing the `decompose = True` parameter to the method will force the WP object to do a full decomposition up to the maximum level of decomposition:

```python
>>> paths = [n.path for n in new_wp.get_leaf_nodes(decompose=True)]
>>> len(paths)
64
>>> for i, path in enumerate(paths):
...     if (i+1) % 8 == 0:
...         print(path)
...     else:
...         try:
...             print(path, end=' ')
```
Lazy evaluation:

Note: This section is for demonstration of pywt internals purposes only. Do not rely on the attribute access to nodes as presented in this example.

```python
>>> x = numpy.array([[1, 2, 3, 4, 5, 6, 7, 8]] * 8)
>>> wp = pywt.WaveletPacket2D(data=x, wavelet='db1', mode='symmetric')
```

1) At first the wp’s attribute `a` is `None`

```python
>>> print(wp.a)
None
```

Remember that you should not rely on the attribute access.

2) During the first attempt to access the node it is computed via decomposition of its parent node (the wp object itself).

```python
>>> print(wp['a'])
a: [[ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]]
```

3) Now the `a` is set to the newly created node:

```python
>>> print(wp.a)
a: [[ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]
 [ 3.  7. 11. 15.]]
```

And so is `wp.d`:

```python
>>> print(wp.d)
d: [[ 0.  0.  0.  0.]
 [ 0.  0.  0.] 0.]
 [ 0.  0.  0.] 0.]
 [ 0.  0.  0.] 0.]
```
5.3.7 Gotchas

PyWavelets utilizes NumPy under the hood. That’s why handling the data containing None values can be surprising. None values are converted to ‘not a number’ (numpy.NaN) values:

```python
>>> import numpy, pywt
>>> x = [None, None]
>>> mode = 'symmetric'
>>> wavelet = 'db1'
>>> cA, cD = pywt.dwt(x, wavelet, mode)
>>> numpy.all(numpy.isnan(cA))
True
>>> numpy.all(numpy.isnan(cD))
True
>>> rec = pywt.idwt(cA, cD, wavelet, mode)
>>> numpy.all(numpy.isnan(rec))
True
```

5.4 Contributing

All contributions including bug reports, bug fixes, new feature implementations and documentation improvements are welcome. Moreover, developers with an interest in PyWavelets are very welcome to join the development team! Please see our guidelines for pull requests for more information.

Contributors are expected to behave in a productive and respectful manner in accordance with our community guidelines and PyWavelets Code of Conduct.

5.4.1 History

PyWavelets started in 2006 as an academic project for a masters thesis on Analysis and Classification of Medical Signals using Wavelet Transforms and was maintained until 2012 by its original developer. In 2013 maintenance was taken over in a new repo by a larger development team - a move supported by the original developer. The repo move doesn’t mean that this is a fork - the package continues to be developed under the name “PyWavelets”, and released on PyPI and Github (see this issue for the discussion where that was decided).

5.5 Development guide

This section contains information on building and installing PyWavelets from source code as well as instructions for preparing the build environment on Windows and Linux.

5.5.1 PyWavelets Code of Conduct

Introduction

This code of conduct applies to all spaces managed by the PyWavelets project, including all public and private mailing lists, issue trackers, wikis, blogs, Twitter, and any other communication channel used by our community. The PyWavelets project does not organise in-person events, however events related to our community should have a code of conduct similar in spirit to this one.
This code of conduct should be honored by everyone who participates in the PyWavelets community formally or informally, or claims any affiliation with the project, in any project-related activities and especially when representing the project, in any role.

This code is not exhaustive or complete. It serves to distill our common understanding of a collaborative, shared environment and goals. Please try to follow this code in spirit as much as in letter, to create a friendly and productive environment that enriches the surrounding community.

This code of conduct was adapted from SciPy’s code of conduct without any substantial modification in terms. We wish to thank the SciPy contributors for their hard work in authoring these terms and in helping develop a community consensus them.

**Specific Guidelines**

We strive to:

1. **Be open.** We invite anyone to participate in our community. We prefer to use public methods of communication for project-related messages, unless discussing something sensitive. This applies to messages for help or project-related support, too; not only is a public support request much more likely to result in an answer to a question, it also ensures that any inadvertent mistakes in answering are more easily detected and corrected.

2. **Be empathetic, welcoming, friendly, and patient.** We work together to resolve conflict, and assume good intentions. We may all experience some frustration from time to time, but we do not allow frustration to turn into a personal attack. A community where people feel uncomfortable or threatened is not a productive one.

3. **Be collaborative.** Our work will be used by other people, and in turn we will depend on the work of others. When we make something for the benefit of the project, we are willing to explain to others how it works, so that they can build on the work to make it even better. Any decision we make will affect users and colleagues, and we take those consequences seriously when making decisions.

4. **Be inquisitive.** Nobody knows everything! Asking questions early avoids many problems later, so we encourage questions, although we may direct them to the appropriate forum. We will try hard to be responsive and helpful.

5. **Be careful in the words that we choose.** We are careful and respectful in our communication and we take responsibility for our own speech. Be kind to others. Do not insult or put down other participants. We will not accept harassment or other exclusionary behaviour, such as:
   - Violent threats or language directed against another person.
   - Sexist, racist, or otherwise discriminatory jokes and language.
   - Posting sexually explicit or violent material.
   - Posting (or threatening to post) other people’s personally identifying information ("doxing").
   - Sharing private content, such as emails sent privately or non-publicly, or unlogged forums such as IRC channel history, without the sender’s consent.
   - Personal insults, especially those using racist or sexist terms.
   - Unwelcome sexual attention.
   - Excessive profanity. Please avoid swearwords; people differ greatly in their sensitivity to swearing.
   - Repeated harassment of others. In general, if someone asks you to stop, then stop.
   - Advocating for, or encouraging, any of the above behaviour.
Diversity Statement

The PyWavelets project welcomes and encourages participation by everyone. We are committed to being a community that everyone enjoys being part of. Although we may not always be able to accommodate each individual’s preferences, we try our best to treat everyone kindly.

No matter how you identify yourself or how others perceive you: we welcome you. Though no list can hope to be comprehensive, we explicitly honour diversity in: age, culture, ethnicity, genotype, gender identity or expression, language, national origin, neurotype, phenotype, political beliefs, profession, race, religion, sexual orientation, socioeconomic status, subculture and technical ability, to the extent that these do not conflict with this code of conduct.

Though we welcome people fluent in all languages, PyWavelets development is conducted in English.

Standards for behaviour in the PyWavelets community are detailed in the Code of Conduct above. Participants in our community should uphold these standards in all their interactions and help others to do so as well (see next section).

Reporting Guidelines

We know that it is painfully common for internet communication to start at or devolve into obvious and flagrant abuse. We also recognize that sometimes people may have a bad day, or be unaware of some of the guidelines in this Code of Conduct. Please keep this in mind when deciding on how to respond to a breach of this Code.

For clearly intentional breaches, report those to the Code of Conduct committee (see below). For possibly unintentional breaches, you may reply to the person and point out this code of conduct (either in public or in private, whatever is most appropriate). If you would prefer not to do that, please feel free to report to the Code of Conduct Committee directly, or ask the Committee for advice, in confidence.

You can report issues to the PyWavelets Code of Conduct committee, at PyWavelets-conduct@googlegroups.com. Currently, the committee consists of:

- Gregory Lee
- Ralf Gommers
- Alexandre de Siqueira

If your report involves any members of the committee, or if they feel they have a conflict of interest in handling it, then they will recuse themselves from considering your report. Alternatively, if for any reason you feel uncomfortable making a report to the committee, then you can also contact:

- Chair of the PyWavelets Steering Committee: Gregory Lee

Incident reporting resolution & Code of Conduct enforcement

This section summarizes the most important points, more details can be found in CoC_reporting_manual.

We will investigate and respond to all complaints. The PyWavelets Code of Conduct Committee and the PyWavelets Steering Committee (if involved) will protect the identity of the reporter, and treat the content of complaints as confidential (unless the reporter agrees otherwise).

In case of severe and obvious breaches, e.g. personal threat or violent, sexist or racist language, we will immediately disconnect the originator from PyWavelets communication channels; please see the manual for details.

In cases not involving clear severe and obvious breaches of this code of conduct, the process for acting on any received code of conduct violation report will be:

1. acknowledge report is received
2. reasonable discussion/feedback

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3. mediation (if feedback didn’t help, and only if both reporter and reportee agree to this)

4. enforcement via transparent decision (see CoC_resolutions) by the Code of Conduct Committee

The committee will respond to any report as soon as possible, and at most within 72 hours.

Endnotes

We are thankful to the SciPy developers for creating the code of conduct we have adapted here.

- Scipy Code of Conduct

The SciPy code of conduct was in turn inspired by the following documents:

- The Apache Foundation Code of Conduct
- The Contributor Covenant
- Jupyter Code of Conduct
- Open Source Guides - Code of Conduct

5.5.2 Preparing Windows build environment

To start developing PyWavelets code on Windows you will have to install a C compiler and prepare the build environment.

Installing Windows SDK C/C++ compiler

Depending on your Python version, a different version of the Microsoft Visual C++ compiler will be required to build extensions. The same compiler that was used to build Python itself should be used.

For Python 3.7 or 3.8 it will be MSVC 2015.

The MSVC version should be printed when starting a Python REPL, and can be checked against the note below:

**Note:** For reference:

- the **MSC v.1500** in the Python version string is Microsoft Visual C++ 2008 (Microsoft Visual Studio 9.0 with msvcr90.dll runtime)
- **MSC v.1600** is MSVC 2010 (10.0 with msvcr100.dll runtime)
- **MSC v.1700** is MSVC 2012 (11.0)
- **MSC v.1800** is MSVC 2013 (12.0)
- **MSC v.1900** is MSVC 2015 (14.0)


There are several ISO images on the site, so just grab the one that is suitable for your platform:

- **GRMSDK_EN_DVD.iso** for 32-bit x86 platform
• GRMSDKX_EN_DVD.iso for 64-bit AMD64 platform (AMD64 is the codename for 64-bit CPU architecture, not the processor manufacturer)

After installing the SDK and before compiling the extension you have to configure some environment variables.

For 32-bit build execute the `util/setenv_build32.bat` script in the cmd window:

```batch
rem Configure the environment for 32-bit builds.
rem Use "vcvars32.bat" for a 32-bit build.
"C:\Program Files (x86)\Microsoft Visual Studio 9.0\VC\bin\vcvars32.bat"
rem Convince setup.py to use the SDK tools.
set MSSdk=1
setenv /x86 /release
set DISTUTILS_USE_SDK=1
```

For 64-bit use `util/setenv_build64.bat`:

```batch
rem Configure the environment for 64-bit builds.
rem Use "vcvars32.bat" for a 32-bit build.
"C:\Program Files (x86)\Microsoft Visual Studio 9.0\VC\bin\vcvars64.bat"
rem Convince setup.py to use the SDK tools.
set MSSdk=1
setenv /x64 /release
set DISTUTILS_USE_SDK=1
```

See also [http://wiki.cython.org/64BitCythonExtensionsOnWindows](http://wiki.cython.org/64BitCythonExtensionsOnWindows).

**MinGW C/C++ compiler**

MinGW distribution can be downloaded from [http://sourceforge.net/projects/mingwbuilds/](http://sourceforge.net/projects/mingwbuilds/).

In order to change the settings and use MinGW as the default compiler, edit or create a Distutils configuration file `c:\Python2*\Lib\distutils\distutils.cfg` and place the following entry in it:

```
[build]
compiler = mingw32
```

You can also take a look at Cython’s “Installing MinGW on Windows” page at [http://wiki.cython.org/InstallingOnWindows](http://wiki.cython.org/InstallingOnWindows) for more info.

**Note:** Python 2.7/3.2 distutils package is incompatible with the current version (4.7+) of MinGW (MinGW dropped the `-mno-cygwin` flag, which is still passed by distutils).

To use MinGW to compile Python extensions you have to patch the `distutils/cygwinccompiler.py` library module and remove every occurrence of `-mno-cygwin`.

See [http://bugs.python.org/issue12641](http://bugs.python.org/issue12641) bug report for more information on the issue.

**Next steps**

After completing these steps continue with *Installing build dependencies.*
5.5.3 Preparing Linux build environment

There is a good chance that you already have a working build environment. Just skip steps that you don’t need to execute.

Installing basic build tools

Note that the example below uses aptitude package manager, which is specific to Debian and Ubuntu Linux distributions. Use your favourite package manager to install these packages on your OS.

```
aptitude install build-essential gcc python-dev git-core
```

Next steps

After completing these steps continue with Installing build dependencies.

5.5.4 Installing build dependencies

Setting up Python virtual environment

A good practice is to create a separate Python virtual environment for each project. If you don’t have virtualenv yet, install and activate it using:

```
curl -O https://raw.github.com/pypa/virtualenv/master/virtualenv.py
python virtualenv.py <name_of_the_venv>
. <name_of_the_venv>/bin/activate
```

Installing Cython

Use pip (http://pypi.python.org/pypi/pip) to install Cython:

```
pip install Cython
```

Installing numpy

Use pip to install numpy:

```
pip install numpy
```

Numpy can also be obtained via scientific python distributions such as:

- Anaconda
- Enthought Canopy
- Python(x,y)

Note: You can find binaries for 64-bit Windows on http://www.lfd.uci.edu/~gohlke/pythonlibs/.
Installing Sphinx

Sphinx is a documentation tool that converts reStructuredText files into nicely looking html documentation. Install it with:

```
pip install Sphinx
```

numpydoc is used to format the API documentation appropriately. Install it via:

```
pip install numpydoc
```

### 5.5.5 Building and installing PyWavelets

#### Installing from source code

Go to https://github.com/PyWavelets/pywt GitHub project page, fork and clone the repository or use the upstream repository to get the source code:

```
git clone https://github.com/PyWavelets/pywt.git PyWavelets
```

Activate your Python virtual environment, go to the cloned source directory and type the following commands to build and install the package:

```
python setup.py build
python setup.py install
```

To verify the installation run the following command:

```
python setup.py test
```

To build docs:

```
cd doc
make html
```

#### Installing a development version

You can also install directly from the source repository:

```
pip install -e git+https://github.com/PyWavelets/pywt.git#egg=PyWavelets
```

or:

```
pip install PyWavelets==dev
```

#### Installing a regular release from PyPi

A regular release can be installed with pip or easy_install:

```
pip install PyWavelets
```
5.5.6 Testing

Continous integration with Travis-CI

The project is using Travis-CI service for continuous integration and testing.

Current build status is:

[build passing]

If you are submitting a patch or pull request please make sure it does not break the build.

Running tests locally

Tests are implemented with pytest, so use one of:

$ pytest --pyargs pywt -v

There are also older doctests that can be run by performing the following from the root of the project source.

$ python pywt/tests/test_doc.py $ cd doc $ make doctest

Additionally the examples in the demo subfolder can be checked by running:

$ python util/refguide_check.py

Note: doctests require Matplotlib in addition to the usual dependencies.

Running tests with Tox

There’s also a config file for running tests with Tox (pip install tox). To for example run tests for Python 3.7
and 3.8 use:

tox -e py37,py38

For more information see the Tox documentation.

5.5.7 Guidelines for Releasing PyWavelets

The following are guidelines for preparing a release of PyWavelets. The notation vX.X.X in the commands below
would be replaced by the actual release number.

Updating the release notes

Prior to the release, make sure the release notes are up to date. The author lists can be generated via:

python ./util/authors.py vP.P.P.

where vP.P.P is the previous release number.

The lists of issues closed and PRs merged can be generated via (script requires Python 2.X to run):

python ./util/gh_lists.py vX.X.X
Tag the release

Change `ISRELEASED` to `True` in `setup.py` and commit.

Tag the release via:

```
git tag -s vX.X.X
```

Then push the `vX.X.X` tag to the PyWavelets GitHub repo.

**Build Windows, OS X and Linux wheels and upload to PyPI**

Pushing the `vX.X.X` tag to the repository will kick off automated build and deployment of the wheels to PyPI. The wheel builds proceed via GitHub Actions and their status can be checked by going to the Actions tab on GitHub.

In the event that the automated deployment fails, the built wheels can be downloaded via the GitHub Actions artifacts and then uploaded manually using `twine` as described below.

**Create the source distribution**

The automated wheel build process should also automatically upload the `sdist` to PyPI. In the event that automated upload of the `sdist` fails, please proceed in generating and uploading it manually as described in this section.

Remove untracked files and directories with `git clean`. *Warning: this will delete files & directories that are not under version control so you may want to do a dry run first by adding `-n`, so you can see what will be removed:*

```
git clean -xfdn
```

Then run without `-n`:

```
git clean -xfd
```

Create the source distribution files via:

```
python setup.py sdist --formats=gztar,zip
```

**Upload the release to PyPI**

These instructions cover how to upload wheels and source distributions to PyPI in the event that the automated deployment fails. The binary Windows wheels downloaded from GitHub Actions (see above) should also be placed into the `/dist` subfolder along with the `sdist` archives.

The wheels and source distributions created above can all be securely uploaded to `pypi.python.org` using `twine`:

```
twine upload -s dist/*
```

Note that the documentation on ReadTheDocs (http://pywavelets.readthedocs.org) will have been automatically generated, so no actions need to be taken for documentation.

**Update conda-forge**

This is an autotick bot run by conda-forge that is likely to autodetect the new PyPI release and autogenerate a PR for you that will update the PyWavelets feedstock for conda-forge. If this automated PR does not appear, you will need to send a PR with the new version number and `sha256` hash of the source release to https://github.com/conda-forge/pywavelets-feedstock.

5.5. Development guide
Create the release on GitHub

On the project’s GitHub page, click the releases tab and then press the “Draft a new release” button to create a release from the appropriate tag.

Announcing the release

Send release announcements to:

- pywavelets@googlegroups.com
- python-announce-list@python.org
- scipy-user@python.org

Prepare for continued development

Increment the version number in setup.py and change ISRELEASED to False.

Prepare new release note files for the upcoming release:

```
$ git add doc/release/X.X.X-notes.rst
$ git add doc/source/release.X.X.X.rst
```

And add `release.X.X.X` to the list in `doc/source/releasenotes.rst`

5.5.8 Something not working?

If these instructions are not clear or you need help setting up your development environment, go ahead and ask on the PyWavelets discussion group at http://groups.google.com/group/pywavelets or open a ticket on GitHub.

5.6 Release Notes

5.6.1 PyWavelets 0.3.0 Release Notes

Contents

- PyWavelets 0.3.0 Release Notes
  - New features
    - Test suite
    - n-D Inverse Discrete Wavelet Transform
  - Thresholding
  - Backwards incompatible changes
  - Other changes
  - Authors
    - Issues closed for v0.3.0
PyWavelets 0.3.0 is the first release of the package in 3 years. It is the result of a significant effort of a growing development team to modernize the package, to provide Python 3.x support and to make a start with providing new features as well as improved performance. A 0.4.0 release will follow shortly, and will contain more significant new features as well as changes/deprecations to streamline the API.

This release requires Python 2.6, 2.7 or 3.3-3.5 and NumPy 1.6.2 or greater.

Highlights of this release include:

- Support for Python 3.x (>=3.3)
- Added a test suite (based on nose, coverage up to 61% so far)
- Maintenance work: C style complying to the Numpy style guide, improved templating system, more complete docstrings, pep8/pyflakes compliance, and more.

New features

Test suite

The test suite can be run with `nose tests pywt` or with:

```python
>>> import pywt
>>> pywt.test()
```

n-D Inverse Discrete Wavelet Transform

The function `pywt.idwtn`, which provides n-dimensional inverse DWT, has been added. It complements `idwt`, `idwt2` and `dwtn`.

Thresholding

The function `pywt.threshold` has been added. It unifies the four thresholding functions that are still provided in the `pywt.thresholding` namespace.

Backwards incompatible changes

None in this release.

Other changes

Development has moved to a new repo. Everyone with an interest in wavelets is welcome to contribute!

Building wheels, building with `python setup.py develop` and many other standard ways to build and install PyWavelets are supported now.
Authors

- Ankit Agrawal +
- François Boulogne +
- Ralf Gommers +
- David Menéndez Hurtado +
- Gregory R. Lee +
- David McInnis +
- Helder Oliveira +
- Filip Wasilewski
- Kai Wohlfahrt +

A total of 9 people contributed to this release. People with a “+” by their names contributed a patch for the first time. This list of names is automatically generated, and may not be fully complete.

Issues closed for v0.3.0

- #3: Remove numerix compat layer
- #4: Add single code base Python 3 support
- #5: PEP8 issues
- #6: Migrate tests to nose
- #7: Expand test coverage without Matlab to a reasonable level
- #8: Replace custom C templates by Numpy’s templating system
- #9: Replace Cython templates by fused types
- #10: Replace use of __array_interface__ with Cython’s memoryviews
- #11: Format existing docstrings in numpydoc format.
- #12: Complete docstrings, they’re quite sparse right now
- #13: Reorganize source tree
- #24: doc/source/regression should be moved
- #27: Broken test: test.swt_decomposition
- #28: Install issue, no module tools.six
- #29: wp.update fails after removal of nodes
- #32: wp.update fails on 2D
- #34: Wavelet string attributes shouldn’t be bytes in Python 3
- #35: Re-enable float32 support
- #36: wavelet instance vs string
- #40: Test with Numpy 1.8rc1
- #45: demos should be updated and integrated in docs
- #60: Moving pywt forward faster
• #61: issues to address in moving towards 0.3.0
• #71: BUG: _pywt.downcoef always returns level=1 result

**Pull requests for v0.3.0**

• #1: travis: check all branches + fix URL
• #17: [DOC] doctrings for multilevel functions
• #18: DOC: format -> functions.py
• #20: MAINT: remove unnecessary zero() copy()
• #21: Doc wavelet_packets
• #22: Minor doc fixes
• #25: TEST: remove useless functions and use numpy instead
• #26: Merge most recent work
• #30: Adding test for wp.rst
• #41: Change to Numpy templating system
• #43: MAINT: update six.py to not use lazy loading.
• #49: Taking on API Issues
• #50: Add idwtn
• #53: readme updated with info related to Py3 version
• #63: Remove six
• #65: Thresholding
• #70: MAINT: PEP8 fixes
• #72: BUG: fix _downcoef for level > 1
• #73: MAINT: documentation and metadata update for repo fork
• #74: STY: fix pep8/pyflakes issues
• #77: MAINT: raise ValueError if data given to dwt or idwt is not 1D...

### 5.6.2 PyWavelets 0.4.0 Release Notes

**Contents**

- *PyWavelets 0.4.0 Release Notes*
  - New features
    - 1D and 2D inverse stationary wavelet transforms
    - Faster 2D and nD wavelet transforms
    - Complex floating point support
    - nD implementation of the multilevel DWT and IDWT
PyWavelets 0.4.0 is the culmination of 6 months of work. In addition to several new features, some changes and deprecations have been made to streamline the API.

This release requires Python 2.6, 2.7 or 3.3-3.5 and NumPy 1.6.2 or greater.

Highlights of this release include:

- 1D and 2D inverse stationary wavelet transforms
- Substantially faster 2D and nD discrete wavelet transforms
- Complex number support
- nD versions of the multilevel DWT and IDWT

New features

1D and 2D inverse stationary wavelet transforms

1D (iswt) and 2D (iswt2) inverse stationary wavelet transforms were added. These currently only support even length inputs.

Faster 2D and nD wavelet transforms

The multidimensional DWT and IDWT code was refactored and is now an order of magnitude faster than in previous releases. The following functions benefit: dwt2, idwt2, dwtn, idwtn.

Complex floating point support

64 and 128-bit complex data types are now supported by all wavelet transforms.

nD implementation of the multilevel DWT and IDWT

The existing 1D and 2D multilevel transforms were supplemented with an nD implementation.
Wavelet transforms can be applied along a specific axis/axes

All wavelet transform functions now support explicit specification of the axis or axes upon which to perform the transform.

Example Datasets

Two additional 2D grayscale images were added (camera, ascent). The previously existing 1D ECG data (ecg) and the 2D aerial image (aero) used in the demos can also now be imported via functions defined in `pywt.data` (e.g. `camera = pywt.data.camera()`)

Deprecated features

A number of functions have been renamed, the old names are deprecated and will be removed in a future release:

- `intwave`, renamed to `integrate_wavelet`
- `centrfrq`, renamed to `central_frequency`
- `scal2frq`, renamed to `scale2frequency`
- `orthfilt`, renamed to `orthogonal_filter_bank`

Integration of general signals (i.e. not wavelets) with `integrate_wavelet` is deprecated.

The `MODES` object and its attributes are deprecated. The new name is `Modes`, and the attribute names are expanded:

- `zpd`, renamed to `zero`
- `cpd`, renamed to `constant`
- `spi`, renamed to `smooth`
- `sym`, renamed to `symmetric`
- `ppd`, renamed to `periodic`
- `per`, renamed to `periodization`

Backwards incompatible changes

`idwt` no longer takes a `correct_size` parameter. As a consequence, `idwt2` inputs must match exactly in length.

For multilevel transforms, where arrays differing in size by one element may be produced, use the `waverec` functions from the `multilevel` module instead.

Bugs Fixed

Float32 inputs were not always respected. All transforms now return float32 outputs when called using float32 inputs.

Incorrect detail coefficients were returned by `downcoef` when `level > 1`.

Other changes

Much of the API documentation is now autogenerated from the corresponding function docstrings. The numpydoc sphinx extension is now needed to build the documentation.
Authors

- Thomas Arildsen +
- François Boulogne
- Ralf Gommers
- Gregory R. Lee
- Michael Marino +
- Aaron O’Leary +
- Daniele Tricoli +
- Kai Wohlfahrt

A total of 8 people contributed to this release. People with a “+” by their names contributed a patch for the first time. This list of names is automatically generated, and may not be fully complete.

Issues closed for v0.4.0

- #46: Independent test comparison
- #95: Simplify Matlab tests
- #97: BUG: erroneous detail coefficients returned by downcoef with...
- #140: demo/dwt_signal_decomposition.py : TypeError: object of type...
- #141: Documentation needs update: ImportError: cannot import name ‘multilevel’

Pull requests for v0.4.0

- #55: [RFC] Api changes
- #59: Refactor convolution.c.src
- #64: MAINT: make LH, HL variable names in idwt2 consistent with dwt2
- #67: ENH: add wavedecn and waverecn functions
- #68: ENH: Faster dwtn and idwtn
- #88: DOC minor edit about possible naming
- #93: Added implementation of iswt and iswt2
- #98: fix downcoef detail coefficients for level > 1
- #99: complex support in all dwt and idwt related functions
- #100: replace mlabwrap with python-matlab-bridge in Matlab tests
- #102: Replace some .src expansion with macros
- #104: Faster idwtn/dwtn
- #106: make sure transforms respect float32 dtype
- #109: DOC: fix broken link in sidebar for html docs.
- #112: Complex fix
• #113: TST: don’t build .exe installers on Appveyor anymore, only wheels.
• #116: [RFC] ENH: Add axis argument to dwt
• #117: MAINT: remove deprecated for loop syntax from Cython code
• #121: Fix typo
• #123: MAINT: remove some unused imports
• #124: switch travis from python 3.5-dev to 3.5
• #130: Add axis argument to multidim
• #138: WIP: Documentation updates for v0.4.0
• #139: Autogenerate function API docs
• #142: fix broken docstring examples in _multilevel.py
• #143: handle None properly in waverec
• #144: Add importable images
• #145: DOC: Document MSVC versions

5.6.3 PyWavelets 0.5.0 Release Notes

Contents

• PyWavelets 0.5.0 Release Notes
  – New features
    • 1D Continuous Wavelet Transforms
    • New discrete wavelets
    • New extension mode: reflect
    • Multilevel DWT Coefficient Handling
    • More C function calls release the GIL
    • Multilevel wavelet transforms along specific axes
    • Faster multilevel stationary wavelet transforms
  – Deprecated features
  – Backwards incompatible changes
  – Bugs Fixed
  – Other changes
  – Authors
    • Issues closed for v0.5.0
    • Pull requests for v0.5.0

PyWavelets is a Python toolbox implementing both discrete and continuous wavelet transforms (mathematical time-frequency transforms) with a wide range of built-in wavelets. C/Cython are used for the low-level routines, enabling high performance. Key Features of PyWavelets are:
• 1D, 2D and nD Forward and Inverse Discrete Wavelet Transform (DWT and IDWT)
• 1D, 2D and nD Multilevel DWT and IDWT
• 1D and 2D Forward and Inverse Stationary Wavelet Transform
• 1D and 2D Wavelet Packet decomposition and reconstruction
• 1D Continuous Wavelet Transform
• When multiple valid implementations are available, we have chosen to maintain consistency with MATLAB™’s Wavelet Toolbox.

PyWavelets 0.5.0 is the culmination of 1 year of work. In addition to several new features, substantial refactoring of the underlying C and Cython code have been made.

This release requires Python 2.6, 2.7 or 3.3-3.5 and NumPy 1.9.1 or greater. This will be the final release supporting Python 2.6 and 3.3.

Highlights of this release include:

• 1D continuous wavelet transforms
• new discrete wavelets added (additional Debauchies and Coiflet wavelets)
• new ‘reflect’ extension mode for discrete wavelet transforms
• faster performance for multilevel forward stationary wavelet transforms (SWT)
• n-dimensional support added to forward SWT
• routines to convert multilevel DWT coefficients to and from a single array
• axis support for multilevel DWT
• substantial refactoring/reorganization of the underlying C and Cython code

New features

1D Continuous Wavelet Transforms

A wide range of continuous wavelets are now available. These include the following:

• Gaussian wavelets (gaus1…”gaus8“)
• Mexican hat wavelet (mexh)
• Morlet wavelet (morl)
• Complex Gaussian wavelets (cgau1…”cgau8“)
• Shannon wavelet (shan)
• Frequency B-Spline wavelet (fbsp)
• Complex Morlet wavelet (cmor)

Also, see the new CWT-related demo: demo/cwt_analysis.py

New discrete wavelets

Additional Debauchies wavelets (db20…”db38“) and Coiflets (coif6…”coif17“) have been added.
New extension mode: reflect

Discrete wavelet transforms support a new extension mode, reflect. This mode pads an array symmetrically, but without repeating the edge value. As an example:

```
<table>
<thead>
<tr>
<th>pad</th>
<th>array</th>
<th>pad</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 3 2</td>
<td>1 2 3 4 5</td>
<td>4 3 2</td>
</tr>
</tbody>
</table>
```

This differs from symmetric, which repeats the values at the boundaries:

```
<table>
<thead>
<tr>
<th>pad</th>
<th>array</th>
<th>pad</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 2 1</td>
<td>1 2 3 4 5</td>
<td>5 4 3</td>
</tr>
</tbody>
</table>
```

Multilevel DWT Coefficient Handling

New routines to convert the coefficients returned by multilevel DWT routines to and from a single n-dimensional array have been added. `pywt.coeffs_to_array` concatenates the output of `wavedec`, `wavedec2` or `wavedecn` into a single numpy array. `pywt.array_to_coeffs` can be used to transform back from a single coefficient array to a format appropriate for `waverec`, `waverec2` or `waverecn`.

More C function calls release the GIL

Cython code calling the wavelet filtering routines (DWT and SWT) now releases the global interpreter lock (GIL) where possible. A potential use case is in speeding up the batch computation of several large DWTs using multi-threading (e.g. via concurrent.futures).

Multilevel wavelet transforms along specific axes

The axis specific transform support introduced in the prior release was extended to the multilevel DWT transforms. All `wavedec*` and `waverec*` routines have a new `axis` (1D) or `axes` (2D, nD) keyword argument. If unspecified the default behaviour is to transform all axes of the input.

Faster multilevel stationary wavelet transforms

Stationary wavelet transforms are now faster when the number of levels is greater than one. The improvement can be very large (multiple orders of magnitude) for transforms with a large number of levels.

Deprecated features

Backwards incompatible changes

A FutureWarning was added to `swt2` and `iswt2` to warn about a pending backwards incompatible change to the order of the coefficients in the list returned by these routines. The actual change will not occur until the next release. Transform coefficients will be returned in descending rather than ascending order. This change is being made for consistency with all other existing multi-level transforms in PyWavelets.
Bugs Fixed

demo/image_blender.py was updated to support the new api of Pillow 3.x
A bug related to size of assumed size_t on some platforms/compilers (e.g. Windows with mingw64) was fixed.
Fix to memory leak in (i)dwt_axis
Fix to a performance regression in idwt and iswt that was introduced in v0.4.0.
Fixed a bug in dwtn and idwtn for data with complex dtype when axes != None.

Other changes

The minimum supported numpy version has been increased to 1.9.1.
Test coverage (including for the Cython and C code) via Codecov was added and the overall test coverage has been improved.
A substantial overhaul of the C extension code has been performed. Custom templating is no longer used. The intention is to make this code easier to maintain and expand in the future.
The Cython code has been split out into a multiple files to hopefully make relevant portions of the wrappers easier to find for future developers.
setup.py now relies on setuptools in all cases (rather than distutils).

Authors

- Jonathan Dan +
- Ralf Gommers
- David Menéndez Hurtado
- Gregory R. Lee
- Holger Nahrstaedt +
- Daniel M. Pelt +
- Alexandre Saint +
- Scott Sievert +
- Kai Wohlfahrt
- Frank Yu +

A total of 10 people contributed to this release. People with a “+” by their names contributed a patch for the first time. This list of names is automatically generated, and may not be fully complete.

Issues closed for v0.5.0

- #48: Continous wavelet transform?
- #127: Reorganize _pywt
- #160: Appveyor failing on recent PRs
- #163: Set up coveralls
- #166: Wavelet coefficients to single array (and vice versa?)
- #177: Fail to install pywt due to the use of index_t which conflict with the definition in /usr/include/sys/types.h on smartos sysmte (open solaris like system)
- #180: Memory leak
- #187: ‘reflect’ signal extension mode
- #189: bump minimum numpy version?
- #191: Upgrade removed Pillow methods
- #196: building in-place for development.
- #200: swt implementation is considerably slower than MATLAB
- #209: broken doctests
- #210: Run doctests in CI setup
- #211: Typo in iswt documentation
- #217: blank_discrete_wavelet does not properly initialize some properties
- #231: I can’t compile pywt

Pull requests for v0.5.0

- #148: Reorganize C v2
- #161: Remove numpy distutils
- #162: fix: iswt/idwt performance regression
- #164: Improved coefficients for db and coif
- #167: Add coverage (codecov.io)
- #168: convert transform coefficients to and from a single n-dimensional array
- #169: Remove templating
- #170: :Always install new pip on Appveyor
- #172: Adding of missing wavelets from the matlab list
- #178: use Index_t instead of index_t
- #179: add axis/axes support to multilevel discrete wavelet transforms
- #181: Fix memory leak
- #182: improve test coverage for _multidim.py and _multilevel.py
- #183: improve coverage for _dwt.py
- #184: fix corner case in coeffs_to_array
- #188: Drop GIL in c_wt calls
- #190: bump minimum numpy to 1.9
- #192: Upgrade to Pillow>=3 api
- #193: ENH: add ‘reflect’ extension mode
- #197: BLD: fix “python setup.py develop”. Closes gh-196
• #198: Choose clz* based on SIZE_MAX
• #201: speedup multi-level swt
• #205: fix dwt/idwt with axes != None and complex data
• #206: DOC: correct typo in iswt docstring
• #207: minor documentation updates
• #208: document coeff_to_array and array_to_coeff
• #214: FIX: update several doctests to reflect the new wavelets added
• #218: FIX: initialize all properties of a blank discrete wavelet
• #219: document coordinate conventions for 2D DWT routines.
• #220: Run doctests on TravisCI
• #221: Documentation for cwt and ContinuousWavelet
• #222: consistent use of double backticks in docs
• #223: add FutureWarning about swt2 coefficient order
• #224: n-dimensional stationary wavelet transform (swtn) and axis support in swt, swt2
• #225: BUG: fix breakage on 32-bit Python.
• #226: DOC: update Copyright statements.
• #227: ENH: add kind keyword to wavelist()
• #228: MAINT: avoid using a builtin as variable name in qmf().
• #229: DOC: add swtn, iswt, iswt2 to the API documentation
• #230: add demo of batch processing via concurrent.futures
• #234: ENH: coeffs_to_array supports axes argument as recently added to wavedec*
• #236: BLD: raise an ImportError if Cython should be installed but isn’t.

5.6.4 PyWavelets 0.5.1 Release Notes

PyWavelets 0.5.1 is a bug-fix release with no new features compared to 0.5.0

Bugs Fixed

In release 0.5.0 the wrong edge mode was used for the following three deprecated modes: ppd, sp1, and per. All deprecated edge mode names are now correctly converted to the corresponding new names.

One-dimensional discrete wavelet transforms did not properly respect the axis argument for complex-valued data. Prior to this release, the last axis was always transformed for arrays with complex dtype. This fix affects dwt, idwt, wavedec, waverec.

Authors

• Gregory R. Lee
Issues closed for v0.5.1

- #245: Keyword “per” for dwt extension mode

Pull requests for v0.5.1

- #244: FIX: dwt, idwt with complex data now pass axis argument properly
- #246: fix bug in deprecated mode name conversion

5.6.5 PyWavelets 0.5.2 Release Notes

PyWavelets 0.5.2 is a bug-fix release with no new features compared to 0.5.1.

Bugs Fixed

The *pywt.data.nino* data reader is now compatible with numpy 1.12. (#273)
The *wp_scalogram.py* demo is now compatible with matplotlib 2.0. (#276)
Fixed a sporadic segmentation fault affecting stationary wavelet transforms of multi-dimensional data. (#289)
`idwtn` now treats coefficients set to None to be treated as zeros (#291). This makes the behavior consistent with its docstring as well as `idwt2`. Previously this raised an error.
The tests are now included when installing from wheels or when running `python setup.py install` (#292)
A bug leading to a potential `RuntimeError` was fixed in `waverec`. This bug only affected transforms where the data was >1D and the transformed axis was not the first axis of the array. (#294).

Authors

- Ralf Gommers
- Gregory R. Lee

Issues closed for v0.5.2

- #280: No tests found from installed version
- #288: RuntimeErrors and segfaults from swt2() in threaded environments
- #290: idwtn should treat coefficients set to None as zeros
- #293: bug in waverec of n-dimensional data when axis != 0

Pull requests for v0.5.2

- #273: fix non-integer index error
- #276: update wp_scalogram demo work with matplotlib 2.0
- #289: fix memory leak in swt_axis
- #291: idwtn should allow coefficients to be set as None
• #292: MAINT: ensure tests are included in wheels
• #294: FIX: shape adjustment in waverec should not assume a transform along . . .
• #295: MAINT: fix readthedocs build issue, update numpy version specifier

5.6.6 PyWavelets 1.0.0 Release Notes

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• PyWavelets 1.0.0 Release Notes
  – New features
    • New 1D test signals
    • C99 complex support
    • complex-valued CWT
    • More flexible specification of some continuous wavelets
    • Fully Separable Discrete Wavelet Transform
    • New thresholding methods
    • New anti-symmetric boundary modes
    • New functions to ravel and unravel wavedecn coefficients
    • New functions to determine multilevel DWT coefficient shapes and sizes
  – Deprecated features
  – Backwards incompatible changes
  – Bugs Fixed
  – Other changes
  – Authors
    • Issues closed for v1.0.0
    • Pull requests for v1.0.0

We are very pleased to announce the release of PyWavelets 1.0. We view this version number as a milestone in the project’s now more than a decade long history. It reflects that PyWavelets has stabilized over the past few years, and is now a mature package which a lot of other important packages depend on. A listing of those package won’t be complete, but some we are aware of are:

• scikit-image - image processing in Python
• imagehash - perceptual image hashing
• pyradiomics - extraction of Radiomics features from 2D and 3D images and binary masks
• tomopy - Tomographic Reconstruction in Python
• SpikeSort - Spike sorting library implemented in Python/NumPy/PyTables
• ODL - operator discretization library
This release requires Python 2.7 or >=3.5 and NumPy 1.9.1 or greater. The 1.0 release will be the last release supporting Python 2.7. It will be a Long Term Support (LTS) release, meaning that we will backport critical bug fixes to 1.0.x for as long as Python itself does so (i.e. until 1 Jan 2020).

**New features**

**New 1D test signals**

Many common synthetic 1D test signals have been implemented in the new function `pywt.data.demo_signals` to encourage reproducible research. To get a list of the available signals, call `pywt.data.demo_signals('list')`. These signals have been validated to match the test signals of the same name from the Wavelab toolbox (with the kind permission of Dr. David Donoho).

**C99 complex support**

The Cython modules and underlying C library can now be built with C99 complex support when supported by the compiler. Doing so improves performance when running wavelet transforms on complex-valued data. On POSIX systems (Linux, Mac OS X), C99 complex support is enabled by default at build time. The user can set the environment variable `USE_C99_COMPLEX` to 0 or 1 to manually disable or enable C99 support at compile time.

**complex-valued CWT**

The continuous wavelet transform, `cwt`, now also accepts complex-valued data.

**More flexible specification of some continuous wavelets**

The continuous wavelets "cmor", "shan" and "fbsp" now let the user specify attributes such as their center frequency and bandwidth that were previously fixed. See more on this in the section on deprecated features.

**Fully Separable Discrete Wavelet Transfrom**

A new variant of the multilevel n-dimensional DWT has been implemented. It is known as the fully separable wavelet transform (FSWT). The functions `fswavedecn fswaverecn` correspond to the forward and inverse transforms, respectively. This differs from the existing `wavedecn` and `waverecn` in dimensions >= 2 in that all levels of decomposition are performed along a single axis prior to moving on to the next.

**New thresholding methods**

`pywt.threshold` now supports non-negative Garotte thresholding (`mode='garotte'`). There is also a new function `pywt.threshold_firm` that implements firm (semi-soft) thresholding. Both of the these new thresholding methods are intermediate between soft and hard thresholding.

**New anti-symmetric boundary modes**

Two new boundary handling modes for the discrete wavelet transforms have been implemented. These correspond to whole-sample and half-sample anti-symmetric boundary conditions (`antisymmetric` and `antireflect`).

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New functions to ravel and unravel wavedecn coefficients

The function ravel_coeffs can be used to ravel all coefficients from wavedec, wavedec2 or wavedecn into a single 1D array. Unraveling back into a list of individual n-dimensional coefficients can be performed by unravel_coeffs.

New functions to determine multilevel DWT coefficient shapes and sizes

The new function wavedecn_size outputs the total number of coefficients that will be produced by a wavedecn decomposition. The function wavedecn_shapes returns full shape information for all coefficient arrays produced by wavedecn. These functions provide the size/shape information without having to explicitly compute a transform.

 Deprecated features

The continuous wavelets with names "cmor", "shan" and "fbsp" should now be modified to include formerly hard-coded attributes such as their center frequency and bandwidth. Use of the bare names “cmor”, “shan” and “fbsp” is now deprecated. For “cmor” (and “shan”), the form of the wavelet name is now “cmorB-C” (“shanB-C”) where B and C are floats representing the bandwidth frequency and center frequency. For “fbsp” the form should now incorporate three floats as in “fbspM-B-C” where M is the spline order and B and C are the bandwidth and center frequencies.

Backwards incompatible changes

Python 2.6, 3.3 and 3.4 are no longer supported.

The order of coefficients returned by swt2 and input to iswt2 have been reversed so that the decomposition levels are now returned in descending rather than ascending order. This makes these 2D stationary wavelet functions consistent with all of the other multilevel discrete transforms in PyWavelets.

For wavedec, wavedec2 and wavedecn, the ability for the user to specify a level that is greater than the value returned by dwt_max_level has been restored. A UserWarning is raised instead of a ValueError in this case.

Bugs Fixed

Assigning new data to the Node or Node2D no longer forces a cast to float64 when the data is one of the other dtypes supported by the dwt(float32, complex64, complex128).

Calling pywt.threshold with mode='soft' now works properly for complex-valued inputs.

A segfault when running multiple swt2 or swtn transforms concurrently has been fixed.

Several instances of deprecated numpy multi-indexing that caused warnings in numpy >=1.15 have been resolved.

The 2d inverse stationary wavelet transform, iswt2, now supports non-square inputs (an unnecessary check for square inputs was removed).

Wavelet packets no longer convert float32 to float64 upon assignment to nodes.

Doctests have been updated to also work with NumPy >= 1.14.

Indexing conventions have been updated to avoid FutureWarnings in NumPy 1.15.

Other changes

Python 3.7 is now officially supported.
Authors

- 0-tree +
- Jacopo Antonello +
- Matthew Brett +
- Saket Choudhary +
- Michael V. DePalatis +
- Daniel Goertzen +
- Ralf Gommers
- Mark Harfouche +
- John Kirkham +
- Dawid Laszuk +
- Gregory R. Lee
- Michel Pelletier +
- Balint Reczey +
- SylvainLan +
- Daniele Tricoli
- Kai Wohlfahrt

A total of 16 people contributed to this release. People with a “+” by their names contributed a patch for the first time. This list of names is automatically generated, and may not be fully complete.

Issues closed for v1.0.0

The following 15 issues were closed for this release.

- #405: New warning appearing
- #397: Make pip install work if numpy is not yet installed
- #396: Allow more levels in wavedec
- #386: Improve documentation for cwt
- #396: Allow more levels in wavedec
- #368: Bug in ISWT2 for non-rectangular arrays
- #363: Crash threading swt2
- #357: reconstruction from array_to_coeff and waverec
- #352: FYI: PyWavelet does not correctly declare setup.py dependency…
- #338: upcoef - TypeError: No matching signature found
- #335: Build issue: PyWavelets does not install from sdist
- #333: user-friendly error messages regarding discrete vs. continuous…
- #326: Allow complex dtype of input
- #316: Test fail in some architectures
• #312: Documentation should suggest using the default conda channel
• #308: incorporate bandwidths into CWT wavelet names for families cmor,….
• #306: dwt_max_levels not enough documentation
• #302: Can’t remove cA and then reconstruct
• #290: idwtn should treat coefficients set to None as zeros
• #288: RuntimeErrors and segfaults from swt2() in threaded environments

Pull requests for v1.0.0

A total of 53 pull requests were merged for this release.
• #248: DOC: sync PyWavelets main descriptions.
• #249: Add pyqtgraph demo for plotting wavelets
• #254: DOC: fix rendering of wavelist docstring
• #255: ENH: improve iswt performance
• #256: ENH: add iswtn (n-dimensional inverse SWT)
• #257: s/addional/additional/
• #260: TST: test OS X build on TravisCI. Closes gh-75.
• #262: avoid some compiler warnings
• #263: MAINT: better exception message for Wavelet(‘continuous_familyname’)
• #264: add ASV (continued)
• #265: MAINT: fix more compiler warnings
• #269: allow string input in dwt_max_level
• #270: DOC: update ISWT documentation
• #272: allow separate wavelet/mode for each axis in routines based on…
• #273: fix non-integer index error
• #275: ENH: use single precision routines for half-precision inputs
• #276: update wp-scalogram demo work with matplotlib 2.0
• #285: Fix spelling typo
• #286: MAINT: Package the license file
• #291: idwtn should allow coefficients to be set as None
• #292: MAINT: ensure tests are included in wheels
• #294: FIX: shape adjustment in waverec should not assume a transform…
• #299: DOC: update outdated scipy-user email address
• #300: ENH: compiling with C99 support (non-MSVC only)
• #303: DOC: better document how to handle omitted coefficients in multilevel…
• #309: Document how max levels are determined for multilevel DWT and…
• #310: parse CWT wavelet names for parameters
• #314: TST: Explicitly align data records in test_byte_offset()
• #317: TST: specify rtol and atol for assert_allclose calls in test_swt_decomposition
• #320: Suggest using default conda channel to install
• #321: BLD: add pyproject.toml file (PEP 518 support).
• #322: support soft thresholding of complex valued data
• #331: Rename to CONTRIBUTING.rst
• #337: provide a more helpful error message for wrong wavelet type
• #339: check for wrong number of dimensions in upcoef and downcoef
• #340: DOC: fix broken link to Airspeed Velocity documentation
• #344: force legacy numpy repr for doctests
• #349: test case for CWT with complex input
• #350: better document the size requirements for swt/swt2/swtn
• #351: Add two new antisymmetric edge modes
• #353: DOC: add citation info to the front page of the docs.
• #354: add firm (semi-soft) and non-negative garotte thresholding
• #355: swt(): inference of level=None to depend on axis
• #356: fix: default level in wavedec2 and wavedecn can be too conservative
• #360: fix Continuous spelling
• #361: AttributeError when using coeffs_to_array
• #362: Fix spelling of continuous globally
• #364: DOC: Explicitly print wavelet name for invalid wavelets
• #367: fix segfault related to parallel SWT
• #369: remove iswt2’s restriction on non-square inputs
• #376: add common 1d synthetic signals
• #377: minor update to demo_signals
• #378: numpy: 1.15 multiindexing warning. targeted fix
• #380: BLD: fix doc build on ReadTheDocs, need matplotlib for plots...
• #381: Fix corner case for small scales in CWT
• #382: avoid FutureWarnings related to multiindexing in Numpy1.15
• #383: adding Community guidelines
• #384: swap swt2 coefficient order (and remove FutureWarnings)
• #387: improve CWT docs
• #391: fix broken link in documentation
• #392: do not force float64 dtype on assignment to Node, Node2D
• #398: MAINT: update .gitignore for files generated during build.

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• #401: Fix failing numpy 1.9.3 build on Travis CI
• #403: Change ValueError to UserWarning when level is > dwt_max_level
• #404: BLD: fix ReadTheDocs build. Outdated NumPy gave a conflict with MPL.
• #410: DOC: rewrite docs front page
• #413: add wavelets.pybytes.com disclaimer

5.6.7 PyWavelets 1.0.1 Release Notes

PyWavelets 1.0.1 is a bug-fix release with no new features compared to 1.0.0.

Bugs Fixed

Key-based assignment of coefficients to a FswavedecnResult object (i.e. via its __setitem__ method) has been fixed.

The order that the individual subband coefficients were stacked by the function `pywt.ravel_coeffs` is now guaranteed to be consistent across all supported Python versions. Explicit alphabetic ordering of subband coefficient names is used for consistent ordering regardless of Python version.

Authors

• Gregory R. Lee

Issues closed for v1.0.1

• #426: Ordering of the coefficients stacked in pywt.ravel_coeffs can vary across Python versions
• #425: error when trying to assign modified coefficients to a FswavedecnResults object

Pull requests for v1.0.1

• #423: fix bug in FswavedecnResult.__setitem__ and improve docstrings
• #427: fix: enforce consistent coefficient order for ravel_coeffs

5.6.8 PyWavelets 1.0.2 Release Notes

PyWavelets 1.0.2 is a bug-fix and maintenance release with no new features compared to 1.0.1.

Bugs Fixed

A bug in `iswtn` when using some combinations of user-specified axes was fixed.

A potential error related to coefficient shape mismatch during WaveletPacket or WaveletPacket2D reconstruction was fixed.
Other Changes

A deprecated import of `Iterable` was fixed.

The spelling of “Garrote” was fixed in the wavelet thresholding documentation. For backwards compatibility with 1.0.0, the incorrect (“garotte”) spelling is also accepted for the `mode` parameter of `pywt.threshold`.

The spelling of “supported” was fixed in one of the `ValueError` messages that can be returned by `pywt.cwt`.

Cython language compatibility has been pinned to `language_level = '2'`. This is in contrast to the `master` branch which is now using `language_level = '3'`. To support this, the minimum supported Cython version has been raised to 0.23.5.

Authors

Four authors contributed PRs for the 1.0.2 release.

Thomas A. Caswell Corey Goldberg Gregory R. Lee Lokesh Ravindranathan

Thanks also goes to Ralf Gommers as a reviewer of most of these.

Issues closed for v1.0.2

- #447: Issue using `pywt.WaveletPacket2D`
- #449: Coefficients arrays must have the same dtype error in `iswt` function
- #460: `iswtn` error when using axes and excluded dim isn’t comply to the level

Pull requests for v1.0.2

- #454: BLD: 1.0.x pin cython language level to ‘2’
- #455: backport of #448 (fix coefficient shape mismatch in WaveletPacket reconstruction)
- #456: MAINT: 1.0.x: Spelling correction
- #457: MAINT: 1.0.x Fix spelling of “Garrote”
- #458: MAINT: 1.0.x Fix deprecated import for `Iterable`
- #464: backport of #448 (fix coefficient shape mismatch in WaveletPacket reconstruction)
- #465: backport of gh-462 (iswtn axis fix)
- #469: MAINT 1.0.x backport #452 (bump minimum supported Cython version)

The backports listed above correspond to the following PRs from the master branch

- #436: Fix deprecated import for `Iterable`
- #438: Fix spelling of “Garrote”
- #446: Spelling correction
- #448: Properly trim wavelet packet node coefficients during reconstruction
- #450: handle mixed dtype coefficients correctly across inverse transforms
- #452: bump minimum supported Cython version
- #462: fix bug in `iswtn` for data of arbitrary shape when using user-specified axes
5.6.9 PyWavelets 1.0.3 Release Notes

PyWavelets 1.0.3 is functionally equivalent to the 1.0.2 release. It was made to add the archive of the JOSS paper to the 1.0.x branch and serve as a reference corresponding to the version of the software reviewed that was peer reviewed.

5.6.10 PyWavelets 1.1.0 Release Notes

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  - Other changes
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We are very pleased to announce the release of PyWavelets 1.1.

This release includes enhanced functionality for both the stationary wavelet transforms (swt, swt2, swtn) as well as the continuous wavelet transform (cwt). In addition, there are a handful of bug fixes as described in more detail below.

This release has dropped Python 2.7 support and now requires Python >= 3.5.

In addition to these changes to the software itself, a paper describing PyWavelets was recently published in The Journal of Open Source Software: https://joss.theoj.org/papers/10.21105/joss.01237

New features

- All swt functions now have a new trim_approx option that can be used to exclude the approximation coefficients from all but the final level of decomposition. This mode makes the output of these functions consistent with the format of the output from the corresponding wavedec functions.

- All swt functions also now have a new norm option that, when set to True and used in combination with trim_approx=True, gives a partition of variance across the transform coefficients. In other words, the sum of the variances of all coefficients is equal to the variance of the original data. This partitioning of variance makes the swt transform more similar to the multiple-overlap DWT (MODWT) described in Percival and Walden’s book, “Wavelet Methods for Time Series Analysis”. (#476)

  A demo of this new swt functionality is available at https://github.com/PyWavelets/pywt/blob/master/demo/swt_variance.py

- The continuous wavelet transform (cwt) now offers an FFT-based implementation in addition to the previous convolution based one. The new method argument can be set to either 'conv' or 'fft' to select between these two implementations. (#490).

- The cwt now also has axis support so that CWTs can be applied in batch along any axis of an n-dimensional array. This enables faster batch transformation of signals. (#509)
Backwards incompatible changes

- When the input to `cwt` is single precision, the computations are now performed in single precision. This was done both for efficiency and to make `cwt` handle dtypes consistently with the discrete transforms in PyWavelets. This is a change from the prior behaviour of always performing the `cwt` in double precision. (#507)

- When using complex-valued wavelets with the `cwt`, the output will now be the complex conjugate of the result that was produced by PyWavelets 1.0.x. This was done to account for a bug described below. The magnitude of the `cwt` coefficients will still match those from previous releases. (#439)

Bugs Fixed

- For a `cwt` with complex wavelets, the results in PyWavelets 1.0.x releases matched the output of Matlab R2012a's `cwt`. However, older Matlab releases like R2012a had a phase that was of opposite sign to that given in textbook definitions of the CWT (Eq. 2 of Torrence and Compo's review article, “A Practical Guide to Wavelet Analysis”). Consequently, the wavelet coefficients were the complex conjugates of the expected result. This was validated by comparing the results of a transform using `cmor1.0-1.0` as compared to the `cwt` implementation available in Matlab R2017b as well as the function `wt.m` from the Lancaster University Physics department’s MODA toolbox. (#439)

- For some boundary modes and data sizes, round-trip `dwt/idwt` can result in an output that has one additional coefficient. Prior to this release, this could cause a failure during `WaveletPacket` or `WaveletPacket2D` reconstruction. These wavelet packet transforms have now been fixed and round-trip wavelet packet transforms always preserve the original data shape. (#448)

- All inverse transforms now handle mixed precision coefficients consistently. Prior to this release some inverse transform raised an error upon encountering mixed precision dtypes in the wavelet subbands. In release 1.1, when the user-provided coefficients are a mixture of single and double precision, all coefficients will be promoted to double precision. (#450)

- A bug that caused a failure for `iswtn` when using user-provided `axes` with non-uniform shape along the transformed axes has been fixed. (#462)

Other changes

- The PyWavelet test suite now uses `pytest` rather than `nose`. (#477)

- Cython code has been updated to use `language_level=3`. (#435)

- PyWavelets has adopted the SciPy Code of Conduct. (#521)

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A total of 10 people contributed to this release. People with a “+” by their names contributed a patch for the first time. This list of names is automatically generated, and may not be fully complete.

**Issues closed for v1.1.0**

• #389: Change test suite from nose to pytest
• #445: Batch load for pywt.cwt
• #449: Coefficients arrays must have the same dtype error in iswt function

**Pull requests for v1.1.0**

• #434: Drop Python 2.7 testing on CI and update docs for Python 3.5+
• #435: set language_level=3 for Cython
• #436: Fix deprecated import for Iterable
• #438: Fix spelling of “Garrote”
• #439: fix the phase of CWT when using complex mother wavelets
• #442: document the numpy.pad equivalent of ‘antireflect’
• #446: Spelling correction
• #448: Properly trim wavelet packet node coefficients during reconstruction
• #450: handle mixed dtype coefficients correctly across inverse transforms
• #462: fix bug in iswtn for data of arbitrary shape when using user-specified
• #463: TST: fix misc. doctest failures (test_doc.py)
• #471: user-friendly error messages about multilevel DWT format
• #476: swt normalization and option to trim the approximation coefficients
• #477: MAINT/TST: update tests to use pytest
• #490: cwt with fft convolution support
• #495: BLD: add missing import of warnings module to setup.py
• #499: register markers for pytest 4.5 compatibility
• #502: fix docstring’s scale2frequency parameter order
• #506: Guard against trying to transform along size 0 axes
• #507: preserve single precision in CWT
• #509: add axis support to cwt
• #510: add demo using swt with norm=True to analyze variance across
• #511: MAINT: split bundled licenses into a separate file
• #514: Small typo in the doc
• #516: Fix docstrings to avoid sphinx warnings
5.6.11 PyWavelets 1.1.1 Release Notes

This release is identical in functionality to 1.1.0.

It fixes setup.py to prevent pip from trying to install from PyPI for Python < 3.5.

5.6.12 PyWavelets 1.2.0 Release Notes

We are very pleased to announce the release of PyWavelets 1.2.

This release has new discrete wavelet transforms features including a series of multiresolution analysis functions (details below).

PyWavelets has dropped support for Python 3.5 and 3.6 and now supports Python 3.7-3.10.

We also now provide aarch64 linux wheels as well as universal2 and arm64 wheels that are compatible with Apple’s M1 processors.

**New features**

- There is a new series of multilevel stationary wavelet transforms (mra, mra2 and mran) suited for multiresolution analysis of 1D, 2D or nD signals, respectively. This MRA analysis is also known as the additive wavelet decomposition because the corresponding inverse functions (imra, imra2 or imran) reconstruct the original signal by simple addition of the components. These are a good alternative to the use of the existing SWT functions when it is important to have features aligned across wavelet scales (see the new demo in demo/mra_vs.swt.py).

- There is now an n-dimensional implementation available for the wavelet packet transforms (see class WaveletPacketND).
Backwards incompatible changes

- The image returned by `pywt.data.camera` has been replaced by a similar, CC0-licensed image because the original image was determined to only be licensed for non-commercial use. Any users who still need the prior camera image for non-commercial use can find it many places online by performing a web search for “cameraman test image”.

Bugs Fixed

- Add input length check in `dwt_single` for reflect modes.
- Demos were updated for compatibility with recent Matplotlib versions.
- Removed deprecated import from `imp`.

Other changes

- PyWavelets has dropped support for Python 3.5 and 3.6 in this release.

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A total of 6 people contributed to this release. People with a “+” by their names contributed a patch for the first time. This list of names is automatically generated, and may not be fully complete.

Issues closed for v1.2

- #235: Build issue: cython not recognized
- #545: Request: add __reduce__ method for WaveletPacket object
- #554: imp is deprecated

Pull requests for v1.2

- #393: Wavelet packets: extend to nD and support subsets of the axes
- #525: WIP: test against Python 3.8
- #527: Multiresolution Analysis
- #540: remove unused distutils import from __init__.py
- #546: ENH: make Wavelet, WaveletPacket, WaveletPacket2D and ContinuousWavelet...
• #552: Cython 3.0 compatibility: explicitly call import_array()
• #559: MAINT: bump minimum Python version to 3.7
• #567: Documentation about trim_approx in.swt2 is wrong
• #571: improve documentation of padding in coeffs_to_array
• #572: Replace camera image with a CC0-licensed equivalent
• #573: switch from terrify to multibuild for OS X setup on Travis
• #578: Add Python 3.9 to CI and classifiers
• #584: MAINT: remove use of deprecated np.int/float aliases
• #598: Add input length check in dwt_single for reflect modes
• #601: Add wheel building and deployment via GitHub Actions and cibuildwheel
• #604: MAINT: update pyproject.toml and setup.py for Python 3.10
• #608: BLD: update Cython build dependency to recent release and <3.0
• #609: MAINT: fix ‘origin=’image’‘ calls that Matplotlib no longer...
• #610: Update GitHub Actions workflow to build Python 3.10 wheels
• #611: MAINT: fix doc build issues


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