pyunlocbox Documentation

Release 0.2.0

EPFL LTS2

July 29, 2016

Contents

CHAPTER 1

About

PyUNLocBoX is a convex optimization toolbox using proximal splitting methods implemented in Python. It is a free software distributed under the BSD license and is a port of the Matlab UNLocBoX toolbox.

- Code : https://github.com/epfl-lts2/pyunlocbox
- Documentation : http://pyunlocbox.readthedocs.org
- PyPI package : https://pypi.python.org/pypi/pyunlocbox
- Travis continuous integration : https://travis-ci.org/epfl-lts2/pyunlocbox
- UNLocBoX matlab toolbox : http://unlocbox.sourceforge.net

1.1 Features

- Solvers
  - Forward-backward splitting algorithm
  - Douglas-Rachford splitting algorithm
- Proximal operators
  - L1-norm
  - L2-norm
  - Projection on the L2-ball

1.2 Installation

PyUnLocBox is continuously tested with Python 2.6, 2.7, 3.2, 3.3 and 3.4.

System-wide installation:

```
$ pip install pyunlocbox
```

Installation in an isolated virtual environment:

```
$ mkvirtualenv --system-site-packages pyunlocbox
$ pip install pyunlocbox
```

You need virtualenvwrapper to run this command. The `--system-site-packages` option could be useful if you want to use a shared system installation of numpy and matplotlib. Their building and installation require quite some dependencies.

Another way is to manually download from PyPI, unpack the package and install with:

```
$ python setup.py install
```

Execute the project test suite once to make sure you have a working install:

```
$ python setup.py test
```

1.3 Authors

PyUNLocBoX was started in 2014 as an academic project for research purpose at the LTS2 laboratory from EPFL. See our website at http://lts2www.epfl.ch.

Development lead :

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Contributors :

- None yet. Why not be the first ?

CHAPTER 2

Tutorials

The following are some tutorials which show and explain how to use the toolbox to solve some real problems. They goes in increasing degree of difficulty. If you have never used the toolbox before, you are encouraged to follow them in order as they build one upon the other.

2.1 Simple least square problem

This simplistic example is only meant to demonstrate the basic workflow of the toolbox. Here we want to solve a least square problem, i.e. we want the solution to converge to the original signal without any constraint. Lets define this signal by :

```
>>> y = [4, 5, 6, 7]
```

The first function to minimize is the sum of squared distances between the current signal x and the original y. For this purpose, we instantiate an L2-norm object :

```
>>> from pyunlocbox import functions
>>> f1 = functions.norm_l2(y=y)
```

This standard function object provides the `eval()`, `grad()` and `prox()` methods that will be useful to the solver. We can evaluate them at any given point :

```
>>> f1.eval([0, 0, 0, 0])
126
>>> f1.grad([0, 0, 0, 0])
array([ -8, -10, -12, -14])
>>> f1.prox([0, 0, 0, 0], 1)
array([ 2.66666667,  3.33333333,  4.        ,  4.66666667])
```

We need a second function to minimize, which usually describes a constraint. As we have no constraint, we just define a dummy function object by hand. We have to define the `_eval()` and `_grad()` methods as the solver we will use requires it :

```
>>> f2 = functions.func()
>>> f2._eval = lambda x: 0
>>> f2._grad = lambda x: 0
```

Note: We could also have used the *pyunlocbox.functions.dummy* function object.

We can now instantiate the solver object :

```
>>> from pyunlocbox import solvers
>>> solver = solvers.forward_backward()
```

And finally solve the problem :

```
>>> x0 = [0, 0, 0, 0]
>>> ret = solvers.solve([f2, f1], x0, solver, atol=1e-5, verbosity='HIGH')
    func evaluation : 0.000000e+00
    norm_l2 evaluation : 1.260000e+02
INFO: Forward-backward method : FISTA
Iteration 1 of forward_backward :
    func evaluation : 0.000000e+00
    norm_l2 evaluation : 1.400000e+01
    objective = 1.40e+01, relative = 8.00e+00
Iteration 2 of forward_backward :
    func evaluation : 0.000000e+00
    norm_l2 evaluation : 1.555556e+00
    objective = 1.56e+00, relative = 8.00e+00
Iteration 3 of forward_backward :
    func evaluation : 0.000000e+00
    norm_l2 evaluation : 3.293044e-02
    objective = 3.29e-02, relative = 4.62e+01
Iteration 4 of forward_backward :
    func evaluation : 0.000000e+00
    norm_l2 evaluation : 8.780588e-03
    objective = 8.78e-03, relative = 2.75e+00
Iteration 5 of forward_backward :
    func evaluation : 0.000000e+00
    norm_l2 evaluation : 6.391406e-03
    objective = 6.39e-03, relative = 3.74e-01
Iteration 6 of forward_backward :
    func evaluation : 0.000000e+00
    norm_l2 evaluation : 5.713369e-04
    objective = 5.71e-04, relative = 1.02e+01
Iteration 7 of forward_backward :
    func evaluation : 0.000000e+00
    norm_l2 evaluation : 1.726501e-05
    objective = 1.73e-05, relative = 3.21e+01
Iteration 8 of forward_backward :
    func evaluation : 0.000000e+00
    norm_l2 evaluation : 6.109470e-05
    objective = 6.11e-05, relative = 7.17e-01
Iteration 9 of forward_backward :
    func evaluation : 0.000000e+00
    norm_l2 evaluation : 1.212636e-05
    objective = 1.21e-05, relative = 4.04e+00
Iteration 10 of forward_backward :
    func evaluation : 0.000000e+00
    norm_l2 evaluation : 7.460428e-09
    objective = 7.46e-09, relative = 1.62e+03
Solution found after 10 iterations :
    objective function f(sol) = 7.460428e-09
    last relative objective improvement : 1.624424e+03
    stopping criterion : ATOL
```

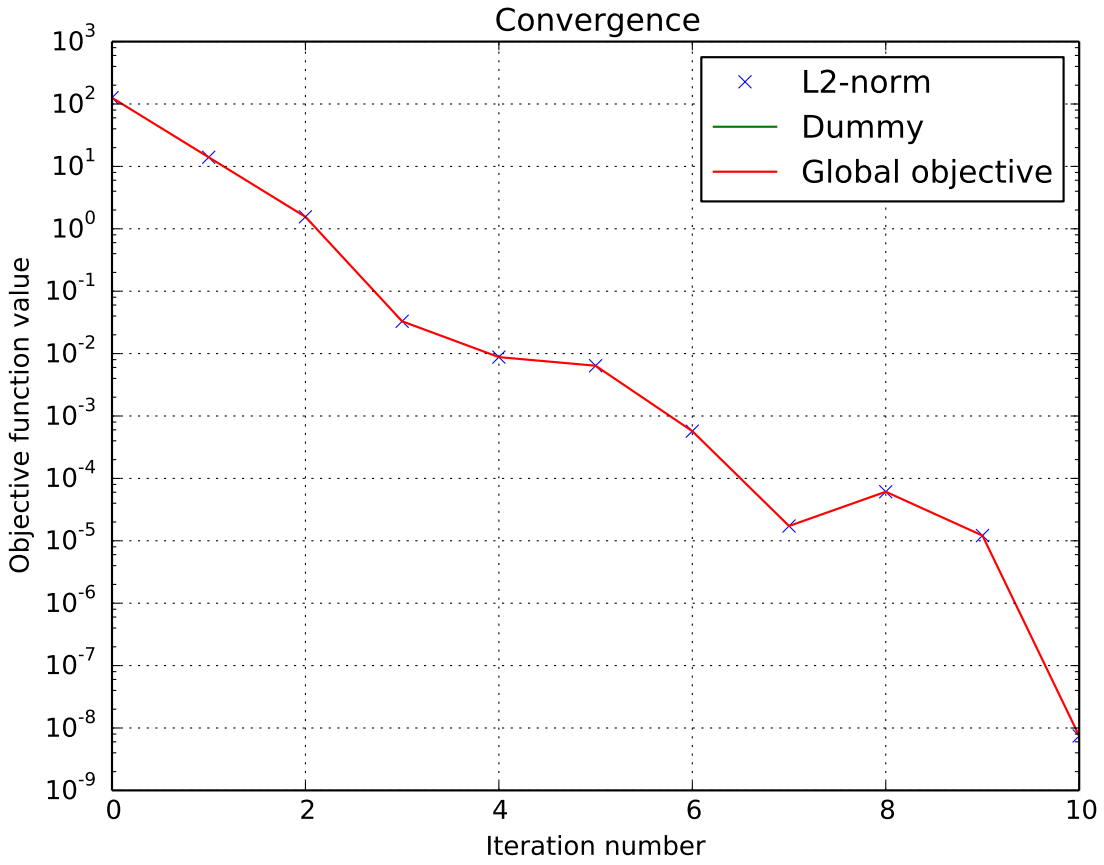
The solving function returns several values, one is the found solution :

```
>>> ret['sol']
array([ 3.99996922,  4.99996153,  5.99995383,  6.99994614])
```

Another one is the value returned by each function objects at each iteration. As we passed two function objects (L2-norm and dummy), the *objective* is a 2 by 11 (10 iterations plus the evaluation at *x0*) `ndarray`. Lets plot a convergence graph out of it :

```
>>> try:
...     import numpy as np
...     import matplotlib, sys
...     cmd_backend = 'matplotlib.use("AGG")'
...     _ = eval(cmd_backend) if 'matplotlib.pyplot' not in sys.modules else 0
...     import matplotlib.pyplot as plt
...     objective = np.array(ret['objective'])
...     _ = plt.figure()
...     _ = plt.semilogy(objective[:, 1], 'x', label='L2-norm')
...     _ = plt.semilogy(objective[:, 0], label='Dummy')
...     _ = plt.semilogy(np.sum(objective, axis=1), label='Global objective')
...     _ = plt.grid(True)
...     _ = plt.title('Convergence')
...     _ = plt.legend(numpoints=1)
...     _ = plt.xlabel('Iteration number')
...     _ = plt.ylabel('Objective function value')
...     _ = plt.savefig('doc/tutorials/simple_convergence.pdf')
...     _ = plt.savefig('doc/tutorials/simple_convergence.png')
... except:
...     pass
```

The below graph shows an exponential convergence of the objective function. The global objective is obviously only composed of the L2-norm as the dummy function object was defined to always evaluate to 0 (`f2._eval = lambda x: 0`).



2.2 Compressed sensing using forward-backward

This tutorial presents a compressed sensing problem solved by the forward-backward splitting algorithm. The problem can be expressed as follow :

$$\arg\min_x \|Ax - y\|^2 + \tau\|x\|_1$$

where *y* are the measurements, *A* is the measurement matrix and *tau* is the regularization parameter.

The number of measurements *M* is computed with respect to the signal size *N* and the sparsity level *K* :

```
>>> N = 5000
>>> K = 100
>>> import numpy as np
>>> M = int(K * max(4, np.ceil(np.log(N))))
>>> print('Number of measurements : %d' % (M,))
Number of measurements : 900
>>> print('Compression ratio : %3.2f' % (float(N)/M,))
Compression ratio : 5.56
```

Note: With the above defined number of measurements, the algorithm is supposed to very often perform a perfect reconstruction.

We generate a random measurement matrix *A* :

```
>>> np.random.seed(1)  # Reproducible results.
>>> A = np.random.standard_normal((M, N))
```

Create the *K* sparse signal *x* :

```
>>> x = np.zeros(N)
>>> I = np.random.permutation(N)
>>> x[I[0:K]] = np.random.standard_normal(K)
>>> x = x / np.linalg.norm(x)
```

Generate the measured signal *y* :

```
>>> y = np.dot(A, x)
```

The first objective function to minimize is defined by

$$f_1(x) = \tau \cdot \|x\|_1$$

which can be expressed by the toolbox L1-norm function object. It can be instantiated as follow, while setting the regularization parameter *tau* :

```
>>> from pyunlocbox import functions
>>> tau = 1.0
>>> f1 = functions.norm_l1(lambda_=tau)
```

The second objective function to minimize is defined by

$$f_2(x) = \|Ax - b\|_2^2$$

which can be expressed by the toolbox L2-norm function object. It can be instantiated as follow :

```
>>> f2 = functions.norm_l2(y=y, A=A)
```

or alternatively as follow :

```
>>> A_ = lambda x: np.dot(A, x)
>>> At_ = lambda x: np.dot(np.transpose(A), x)
>>> f3 = functions.norm_l2(y=y, A=A_, At=At_)
```

Note: In this case the forward and adjoint operators were passed as functions not as matrices.

A third alternative would be to define the function object by hand :

```
>>> f4 = functions.func()
>>> f4._grad = lambda x: 2.0 * np.dot(np.transpose(A), np.dot(A, x) - y)
>>> f4._eval = lambda x: np.linalg.norm(np.dot(A, x) - y)**2
```

Note: The three alternatives to instantiate the function objects (*f2*, *f3* and *f4*) are strictly equivalent and give the exact same results.

Now that the two function objects to minimize (the L1-norm and the L2-norm) are instantiated, we can instantiate the solver object. The step size for optimal convergence is $\frac{1}{\beta}$ where β is the Lipschitz constant of the gradient of *f2*, *f3*, *f4* given by:

$$\beta = 2 \cdot \|A\|_{\text{op}}^2 = 2 \cdot \lambda_{max}(A^* A).$$

To solve this problem, we use the forward-backward splitting algorithm which is instantiated as follow :

```
>>> gamma = 0.5 / np.linalg.norm(A, ord=2)**2
>>> from pyunlocbox import solvers
>>> solver = solvers.forward_backward(method='FISTA', gamma=gamma)
```

Note: A complete description of the constructor parameters and default values is given by the solver object *pyunlocbox.solvers.forward_backward* reference documentation.

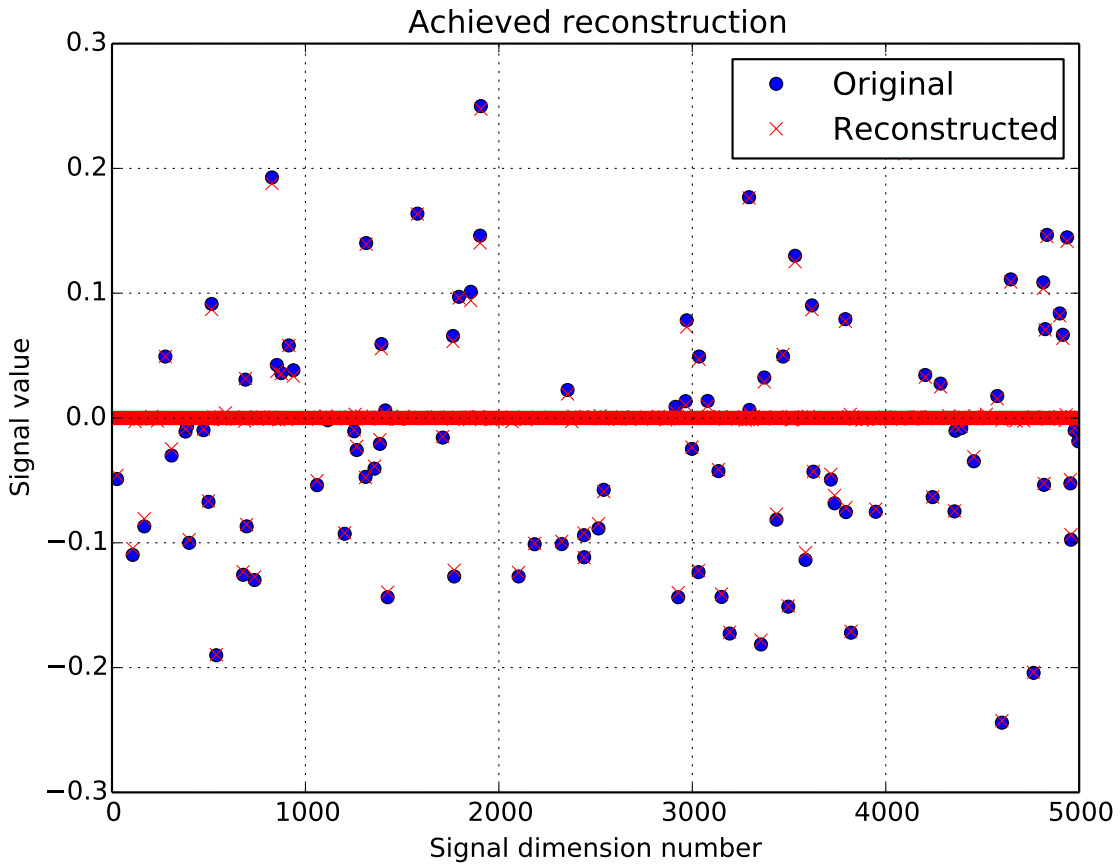
After the instantiations of the functions and solver objects, the setting of a starting point *x0*, the problem is solved by the toolbox solving function as follow :

```
>>> x0 = np.zeros(N)
>>> ret = solvers.solve([f1, f2], x0, solver, rtol=1e-4, maxit=300)
Solution found after 176 iterations :
    objective function f(sol) = 8.221302e+00
    last relative objective improvement : 8.363264e-05
    stopping criterion : RTOL
```

Note: A complete description of the parameters, their default values and the returned values is given by the solving function *pyunlocbox.solvers.solve()* reference documentation.

Let's display the results :

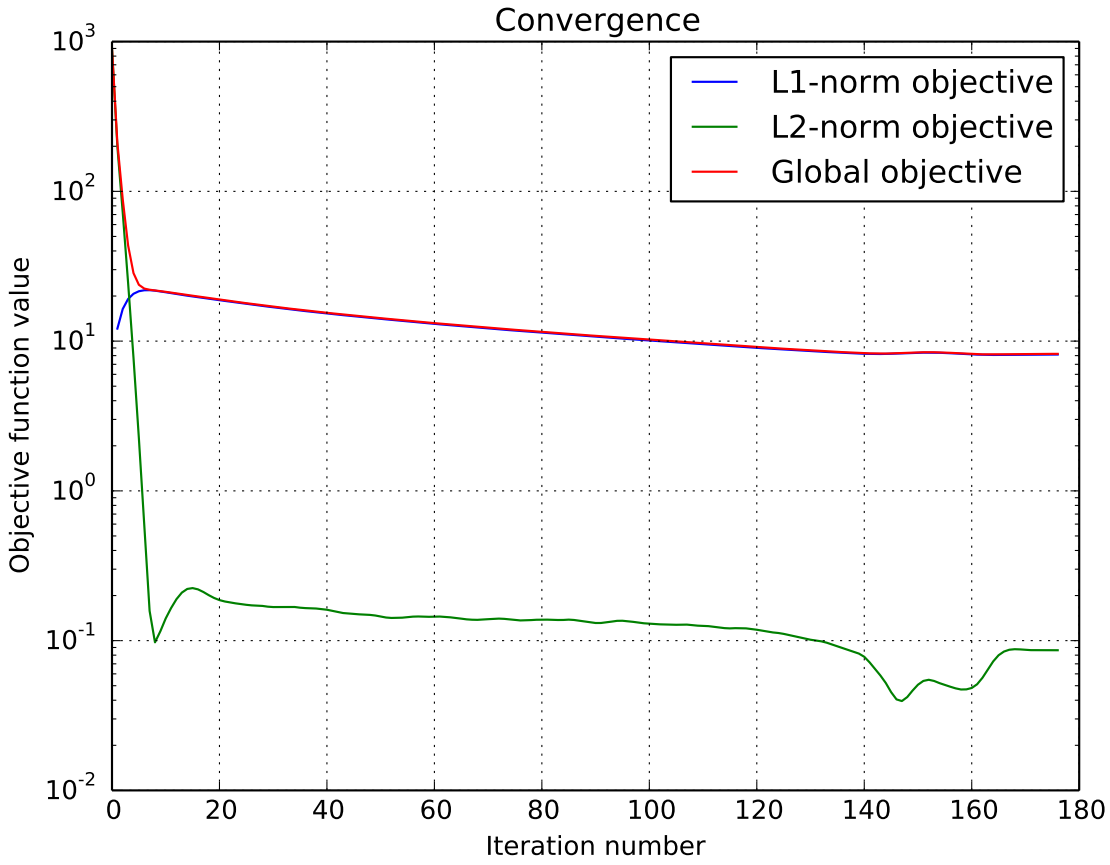
```
>>> try:
...     import matplotlib, sys
...     cmd_backend = 'matplotlib.use("AGG")'
...     _ = eval(cmd_backend) if 'matplotlib.pyplot' not in sys.modules else 0
...     import matplotlib.pyplot as plt
...     _ = plt.figure()
...     _ = plt.plot(x, 'o', label='Original')
...     _ = plt.plot(ret['sol'], 'xr', label='Reconstructed')
...     _ = plt.grid(True)
...     _ = plt.title('Achieved reconstruction')
...     _ = plt.legend(numpoints=1)
...     _ = plt.xlabel('Signal dimension number')
...     _ = plt.ylabel('Signal value')
...     _ = plt.savefig('doc/tutorials/cs_fb_results.pdf')
...     _ = plt.savefig('doc/tutorials/cs_fb_results.png')
... except:
...     pass
```



The above figure shows a good reconstruction which is both sparse (thanks to the L1-norm objective) and close to the measurements (thanks to the L2-norm objective).

Let's display the convergence of the two objective functions :

```
>>> try:
...     objective = np.array(ret['objective'])
...     _ = plt.figure()
...     _ = plt.semilogy(objective[:, 0], label='L1-norm objective')
...     _ = plt.semilogy(objective[:, 1], label='L2-norm objective')
...     _ = plt.semilogy(np.sum(objective, axis=1), label='Global objective')
...     _ = plt.grid(True)
...     _ = plt.title('Convergence')
...     _ = plt.legend()
...     _ = plt.xlabel('Iteration number')
...     _ = plt.ylabel('Objective function value')
...     _ = plt.savefig('doc/tutorials/cs_fb_convergence.pdf')
...     _ = plt.savefig('doc/tutorials/cs_fb_convergence.png')
... except:
...     pass
```



2.3 Compressed sensing using douglas-rachford

This tutorial presents a compressed sensing problem solved by the douglas-rachford splitting algorithm. The problem can be expressed as follow :

$$\arg\min_x \|x\|_1 \qquad \text{such that} \qquad \|Ax - y\|_2 \leq \epsilon$$

where y are the measurements and A is the measurement matrix.

The number of measurements M is computed with respect to the signal size N and the sparsity level K :

```
>>> N = 5000
>>> K = 100
>>> import numpy as np
>>> M = int(K * max(4, np.ceil(np.log(N))))
>>> print('Number of measurements : %d' % (M,))
Number of measurements : 900
>>> print('Compression ratio : %3.2f' % (float(N)/M,))
Compression ratio : 5.56
```

Note: With the above defined number of measurements, the algorithm is supposed to very often perform a perfect reconstruction.

We generate a random measurement matrix *A* :

```
>>> np.random.seed(1)  # Reproducible results.
>>> A = np.random.standard_normal((M, N))
```

Create the *K* sparse signal *x* :

```
>>> x = np.zeros(N)
>>> I = np.random.permutation(N)
>>> x[I[0:K]] = np.random.standard_normal(K)
>>> x = x / np.linalg.norm(x)
```

Generate the measured signal *y* :

```
>>> y = np.dot(A, x)
```

The first objective function to minimize is defined by

$$f_1(x) = \|x\|_1$$

which can be expressed by the toolbox L1-norm function object. It can be instantiated as follow :

```
>>> from pyunlocbox import functions
>>> f1 = functions.norm_l1()
```

The second objective function to minimize is defined by

$$f_2(x) = i_S(x)$$

where $i_S()$ is the indicator function of the set S which is zero if z is in the set and infinite otherwise. The set S is defined by $\left\{z \in \mathbb{R}^N \mid \|A(z) - y\|_2 \leq \epsilon\right\}$. This function can be expressed by the toolbox L2-ball function object which can be instantiated as follow :

```
>>> f2 = functions.proj_b2(epsilon=1e-7, y=y, A=A, tight=False,
... nu=np.linalg.norm(A, ord=2)**2)
```

Now that the two function objects to minimize (the L1-norm and the L2-ball) are instantiated, we can instantiate the solver object. To solve this problem, we use the douglas-rachford splitting algorithm which is instantiated as follow :

```
>>> from pyunlocbox import solvers
>>> solver = solvers.douglas_rachford(gamma=1e-2)
```

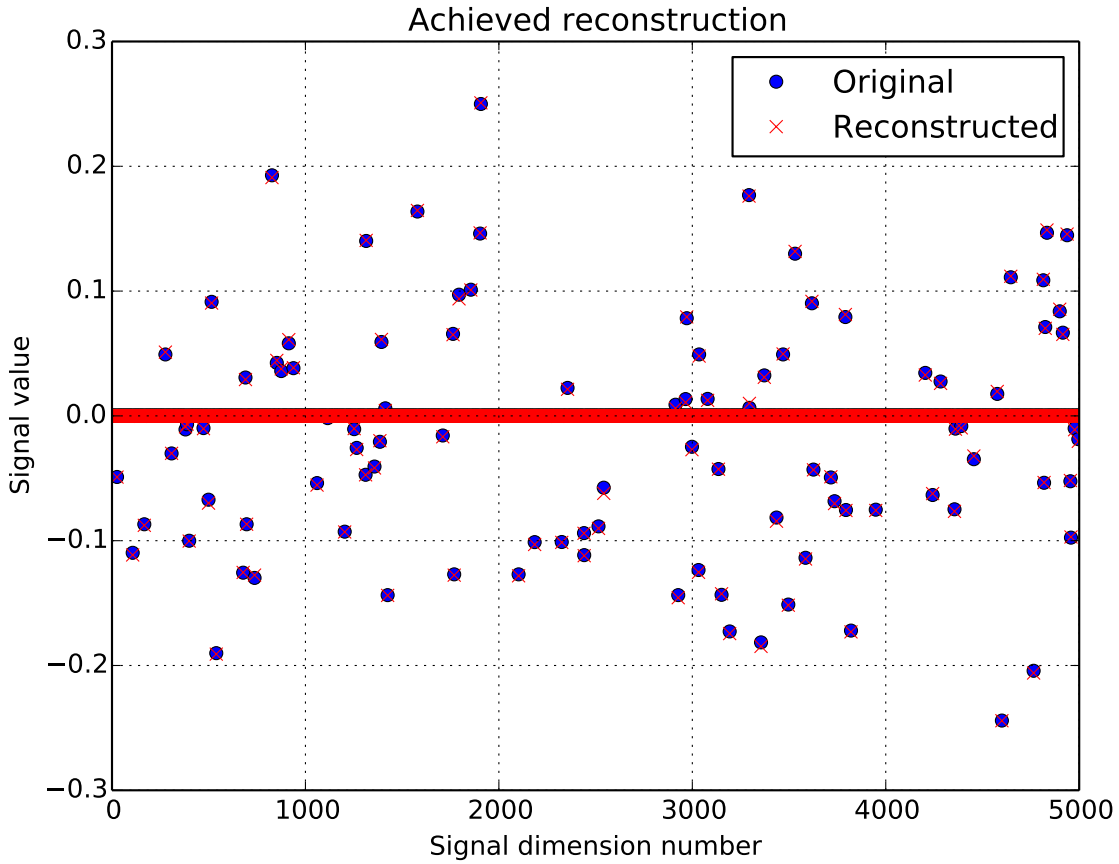
After the instantiations of the functions and solver objects, the setting of a starting point *x0*, the problem is solved by the toolbox solving function as follow :

```
>>> x0 = np.zeros(N)
>>> ret = solvers.solve([f1, f2], x0, solver, rtol=1e-4, maxit=300)
Solution found after 35 iterations :
    objective function f(sol) = 8.508725e+00
    last relative objective improvement : 6.016694e-05
    stopping criterion : RTOL
```

Let's display the results :

```
>>> try:
...     import matplotlib, sys
...     cmd_backend = 'matplotlib.use("AGG")'
...     _ = eval(cmd_backend) if 'matplotlib.pyplot' not in sys.modules else 0
...     import matplotlib.pyplot as plt
...     _ = plt.figure()
...     _ = plt.plot(x, 'o', label='Original')
```

```
...     _ = plt.plot(ret['sol'], 'xr', label='Reconstructed')
...     _ = plt.grid(True)
...     _ = plt.title('Achieved reconstruction')
...     _ = plt.legend(numpoints=1)
...     _ = plt.xlabel('Signal dimension number')
...     _ = plt.ylabel('Signal value')
...     _ = plt.savefig('doc/tutorials/cs_dr_results.pdf')
...     _ = plt.savefig('doc/tutorials/cs_dr_results.png')
... except:
...     pass
```

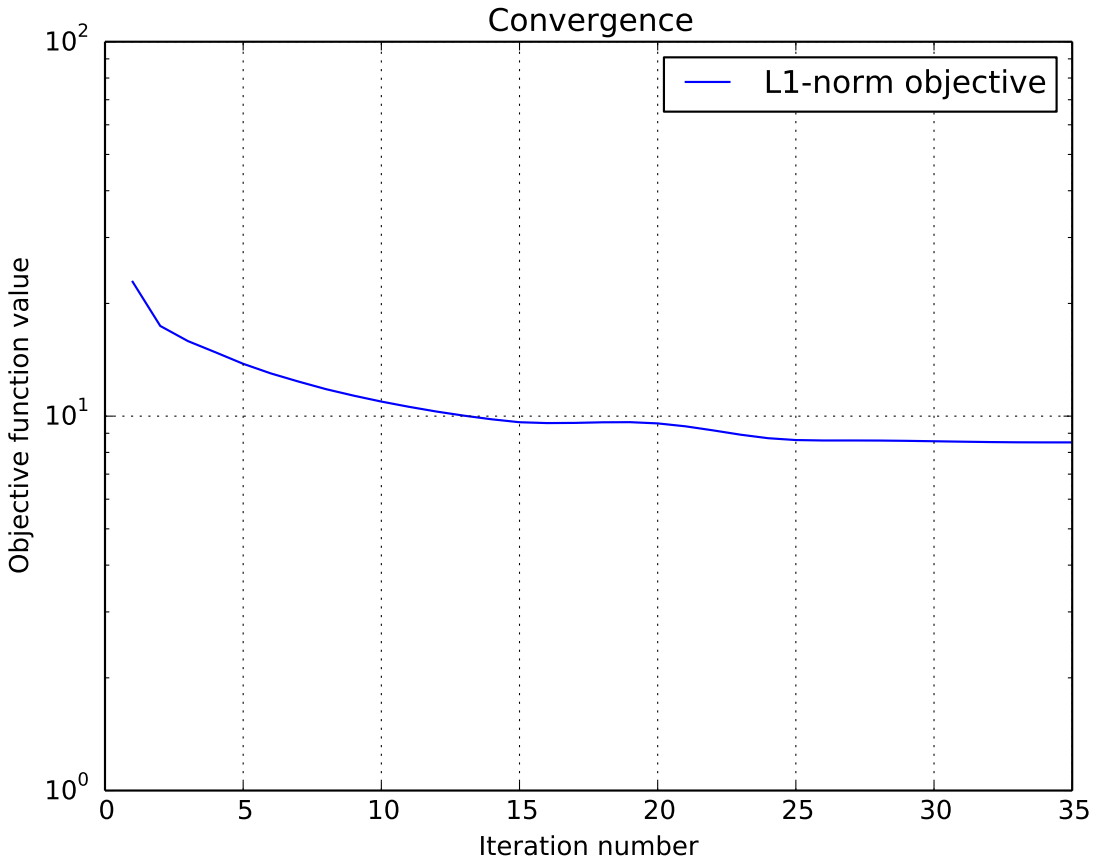


The above figure shows a good reconstruction which is both sparse (thanks to the L1-norm objective) and close to the measurements (thanks to the L2-ball constraint).

Let's display the convergence of the objective function :

```
>>> try:
...     objective = np.array(ret['objective'])
...     _ = plt.figure()
...     _ = plt.semilogy(objective[:, 0], label='L1-norm objective')
...     _ = plt.grid(True)
...     _ = plt.title('Convergence')
...     _ = plt.legend()
...     _ = plt.xlabel('Iteration number')
...     _ = plt.ylabel('Objective function value')
...     _ = plt.savefig('doc/tutorials/cs_dr_convergence.pdf')
...     _ = plt.savefig('doc/tutorials/cs_dr_convergence.png')
```

```
... except:
...     pass
```


Convergence
L1-norm objective
Objective function value
Iteration number
10^2
10^1
10^0
0
5
10
15
20
25
30
35

CHAPTER 3

Reference guide

3.1 Toolbox overview

PyUNLocBoX is a convex optimization toolbox using proximal splitting methods. It is a port of the Matlab UNLocBoX toolbox.

The toolbox is organized around two classes hierarchies : the functions and the solvers. Instantiated functions represent convex functions to optimize. Instantiated solvers represent solving algorithms. The *pyunlocbox.solvers.solve()* solving function takes as parameters a solver object and some function objects to actually solve the optimization problem.

The *pyunlocbox* package is divided into the following modules :

- *pyunlocbox.solvers*: problem solvers, implement the solvers class hierarchy and the solving function
- *pyunlocbox.functions*: functions to be passed to the solvers, implement the functions class hierarchy

Following is a typical usage example who solves an optimization problem composed by the sum of two convex functions. The functions and solver objects are first instantiated with the desired parameters. The problem is then solved by a call to the solving function.

```
>>> import pyunlocbox
>>> f1 = pyunlocbox.functions.norm_l2(y=[4, 5, 6, 7])
>>> f2 = pyunlocbox.functions.dummy()
>>> solver = pyunlocbox.solvers.forward_backward()
>>> ret = pyunlocbox.solvers.solve([f1, f2], [0, 0, 0, 0], solver, atol=1e-5)
Solution found after 10 iterations :
    objective function f(sol) = 7.460428e-09
    last relative objective improvement : 1.624424e+03
    stopping criterion : ATOL
>>> ret['sol']
array([ 3.99996922,  4.99996153,  5.99995383,  6.99994614])
```

3.2 Functions module

3.2.1 Function objects

Interface

class pyunlocbox.functions.**func**(*y=0*, *A=None*, *At=None*, *tight=True*, *nu=1*, *tol=0.001*, *maxit=200*)

Bases: object

This class defines the function object interface.

It is intended to be a base class for standard functions which will implement the required methods. It can also be instantiated by user code and dynamically modified for rapid testing. The instanced objects are meant to be passed to the *pyunlocbox.solvers.solve()* solving function.

Parameters **y** : array_like, optional

Measurements. Default is 0.

A : function or ndarray, optional

The forward operator. Default is the identity, $A(x) = x$. If *A* is an ndarray, it will be converted to the operator form.

At : function or ndarray, optional

The adjoint operator. If *At* is an ndarray, it will be converted to the operator form. If *A* is an ndarray, default is the transpose of *A*. If *A* is a function, default is *A*, $At(x) = A(x)$.

tight : bool, optional

True if *A* is a tight frame, False otherwise. Default is True.

nu : float, optional

Bound on the norm of the operator *A*, i.e. $\|A(x)\|^2 \leq \nu\|x\|^2$. Default is 1.

tol : float, optional

The tolerance stopping criterion. The exact definition depends on the function object, please see the documentation of the considered function. Default is 1e-3.

maxit : int, optional

The maximum number of iterations. Default is 200.

Examples

Let's define a parabola as an example of the manual implementation of a function object :

```
>>> import pyunlocbox
>>> f = pyunlocbox.functions.func()
>>> f._eval = lambda x: x**2
>>> f._grad = lambda x: 2*x
>>> x = [1, 2, 3, 4]
>>> f.eval(x)
array([ 1,  4,  9, 16])
>>> f.grad(x)
array([2, 4, 6, 8])
```

```
>>> f.cap(x)
['EVAL', 'GRAD']
```

cap (*x*)

Test the capabilities of the function object.

Parameters **x** : array_like

The evaluation point. Not really needed, but this function calls the methods of the object to test if they can properly execute without raising an exception. Therefore it needs some evaluation point with a consistent size.

Returns **cap** : list of string

A list of capabilities ('EVAL', 'GRAD', 'PROX').

eval (*x*)

Function evaluation.

Parameters **x** : array_like

The evaluation point.

Returns **z** : float

The objective function evaluated at *x*.

Notes

This method is required by the *pyunlocbox.solvers.solve()* solving function to evaluate the objective function.

grad (*x*)

Function gradient.

Parameters **x** : array_like

The evaluation point.

Returns **z** : ndarray

The objective function gradient evaluated at *x*.

Notes

This method is required by some solvers.

prox (*x*, *T*)

Function proximal operator.

Parameters **x** : array_like

The evaluation point.

T : float

The regularization parameter.

Returns **z** : ndarray

The proximal operator evaluated at *x*.

Notes

This method is required by some solvers.

The proximal operator is defined by $\text{prox}_{f,\gamma}(x) = \arg\min_z \frac{1}{2}\|x - z\|_2^2 + \gamma f(z)$

Dummy function

class pyunlocbox.functions.**dummy**(***kwargs*)
Bases: *pyunlocbox.functions.func*

Dummy function object.

This can be used as a second function object when there is only one function to minimize. It always evaluates as 0.

Examples

```
>>> import pyunlocbox
>>> f = pyunlocbox.functions.dummy()
>>> x = [1, 2, 3, 4]
>>> f.eval(x)
0
>>> f.prox(x, 1)
array([1, 2, 3, 4])
>>> f.grad(x)
array([ 0.,  0.,  0.,  0.])
```

3.2.2 Norm operators class hierarchy

Base class

class pyunlocbox.functions.**norm**(*lambda_=1*, *w=1*, ***kwargs*)
Bases: *pyunlocbox.functions.func*

Base class which defines the attributes of the *norm* objects.

See generic attributes descriptions of the *pyunlocbox.functions.func* base class.

Parameters **lambda_** : float, optional

Regularization parameter λ. Default is 1.

w : array_like, optional

Weights for a weighted norm. Default is 1.

L1-norm

class pyunlocbox.functions.**norm_l1**(***kwargs*)
Bases: *pyunlocbox.functions.norm*

L1-norm function object.

See generic attributes descriptions of the *pyunlocbox.functions.norm* base class. Note that the constructor takes keyword-only parameters.

Notes

- The L1-norm of the vector x is given by $\lambda \|w \cdot (A(x) - y)\|_1$.
- The L1-norm proximal operator evaluated at x is given by $\arg\min_z \frac{1}{2}\|x-z\|_2^2 + \gamma \|w \cdot (A(z) - y)\|_1$ where $\gamma = \lambda \cdot T$. This is simply a soft thresholding.

Examples

```
>>> import pyunlocbox
>>> f = pyunlocbox.functions.norm_l1()
>>> f.eval([1, 2, 3, 4])
10
>>> f.prox([1, 2, 3, 4], 1)
array([ 0.,  1.,  2.,  3.])
```

L2-norm

class pyunlocbox.functions.**norm_l2**(***kwargs*)

Bases: *pyunlocbox.functions.norm*

L2-norm function object.

See generic attributes descriptions of the *pyunlocbox.functions.norm* base class. Note that the constructor takes keyword-only parameters.

Notes

- The squared L2-norm of the vector x is given by $\lambda \|w \cdot (A(x) - y)\|_2^2$.
- The squared L2-norm proximal operator evaluated at x is given by $\arg\min_z \frac{1}{2}\|x-z\|_2^2 + \gamma \|w \cdot (A(z) - y)\|_2^2$ where $\gamma = \lambda \cdot T$.
- The squared L2-norm gradient evaluated at x is given by $2\lambda \cdot At(w \cdot (A(x) - y))$.

Examples

```
>>> import pyunlocbox
>>> f = pyunlocbox.functions.norm_l2()
>>> x = [1, 2, 3, 4]
>>> f.eval(x)
30
>>> f.prox(x, 1)
array([ 0.33333333,  0.66666667,  1.        ,  1.33333333])
>>> f.grad(x)
array([2, 4, 6, 8])
```

3.2.3 Projection operators class hierarchy

Base class

class pyunlocbox.functions.**proj**(*epsilon=0.001*, *method='FISTA'*, ***kwargs*)
Bases: *pyunlocbox.functions.func*

Base class which defines the attributes of the *proj* objects.

See generic attributes descriptions of the *pyunlocbox.functions.func* base class.

Parameters **epsilon** : float, optional

The radius of the ball. Default is 1e-3.

method : {'FISTA', 'ISTA'}, optional

The method used to solve the problem. It can be 'FISTA' or 'ISTA'. Default is 'FISTA'.

L2-ball

class pyunlocbox.functions.**proj_b2**(***kwargs*)
Bases: *pyunlocbox.functions.proj*

L2-ball function object.

This function is the indicator function $i_S(z)$ of the set S which is zero if z is in the set and infinite otherwise. The set S is defined by $\left\{z \in \mathbb{R}^N \mid \|A(z) - y\|_2 \leq \epsilon\right\}$.

See generic attributes descriptions of the *pyunlocbox.functions.proj* base class. Note that the constructor takes keyword-only parameters.

Notes

- The *tol* parameter is defined as the tolerance for the projection on the L2-ball. The algorithm stops if $\frac{\epsilon}{1-tol} \leq \|y - A(z)\|_2 \leq \frac{\epsilon}{1+tol}$.
- The evaluation of this function is zero.
- The L2-ball proximal operator evaluated at x is given by $\arg\min_z \frac{1}{2}\|x-z\|_2^2 + i_S(z)$ which has an identical solution as $\arg\min_z \|x - z\|_2^2$ such that $\|A(z) - y\|_2 \leq \epsilon$. It is thus a projection of the vector x onto an L2-ball of diameter *epsilon*.

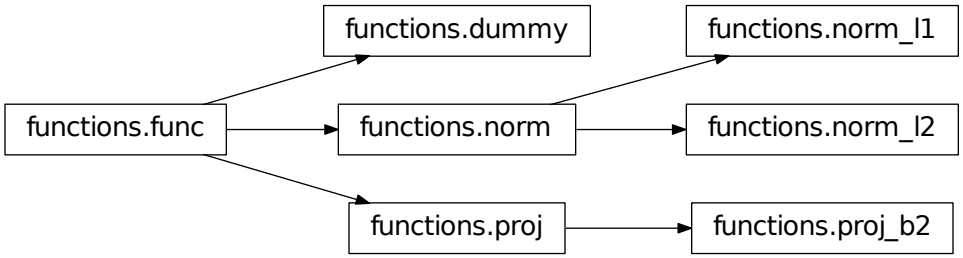
Examples

```
>>> import pyunlocbox
>>> f = pyunlocbox.functions.proj_b2(y=[1, 2])
>>> x = [3, 3]
>>> f.eval(x)
0
>>> f.prox(x, 1)
array([ 1.00089443,  2.00044721])
```

This module implements function objects which are then passed to solvers. The *func* base class defines the interface whereas specialised classes who inherit from it implement the methods. These classes include :

- *dummy*: A dummy function object which returns 0 for the `_eval()`, `_prox()` and `_grad()` methods.

- *norm*: Norm operators base class.
  - *norm_l1*: L1-norm who implements the `_eval()` and `_prox()` methods.
  - *norm_l2*: L2-norm who implements the `_eval()`, `_prox()` and `_grad()` methods.
- *proj*: Projection operators base class.
  - *proj_b2*: Projection on the L2-ball who implements the `_eval()` and `_prox()` methods.



3.3 Solvers module

3.3.1 Solving function

`pyunlocbox.solvers.`**`solve`**(*functions*, *x0*, *solver=None*, *rtol=0.001*, *atol=-inf*, *convergence_speed=-inf*, *maxit=200*, *verbosity='LOW'*)

Solve an optimization problem whose objective function is the sum of some convex functions.

This function minimizes the objective function $f(x) = \sum_{m=0}^{m=M} f_m(x)$, i.e. solves $\arg\min_x f(x)$ for $x \in \mathbb{R}^N$ using whatever algorithm. It returns a dictionary with the found solution and some informations about the algorithm execution.

Parameters **functions** : list of objects

A list of convex functions to minimize. These are objects who must implement the *pyunlocbox.functions.func.eval()* method. The *pyunlocbox.functions.func.grad()* and / or *pyunlocbox.functions.func.prox()* methods are required by some solvers. Note also that some solvers can only handle two convex functions while others may handle more. Please refer to the documentation of the considered solver.

x0 : array_like

Starting point of the algorithm, $x_0 \in \mathbb{R}^N$.

solver : solver class instance, optional

The solver algorithm. It is an object who must inherit from *pyunlocbox.solvers.solver* and implement the `_pre()`, `_algo()` and `_post()` methods. If no solver object are provided, a standard one will be chosen given the number of convex function objects and their implemented methods.

rtol : float, optional

The convergence (relative tolerance) stopping criterion. The algorithm stops if $\left|\frac{n(k-1)-n(k)}{n(k)}\right| < rtol$ where $n(k) = f(x)$ is the objective function at iteration k. Default is 10^{-3}.

atol : float, optional

The absolute tolerance stopping criterion. The algorithm stops if $n(k) < atol$. Default is minus infinity.

convergence_speed : float, optional

The minimum tolerable convergence speed of the objective function. The algorithm stops if n(k-1) - n(k) < *convergence_speed*. Default is minus infinity (i.e. the objective function may even increase).

maxit : int, optional

The maximum number of iterations. Default is 200.

verbosity : {'NONE', 'LOW', 'HIGH', 'ALL'}, optional

The log level : `'NONE'` for no log, `'LOW'` for resume at convergence, `'HIGH'` for info at all solving steps, `'ALL'` for all possible outputs, including at each steps of the proximal operators computation. Default is `'LOW'`.

Returns **sol** : ndarray

The problem solution.

solver : str

The used solver.

niter : int

The number of iterations.

time : float

The execution time in seconds.

eval : float

The final evaluation of the objective function $f(x)$.

crit : {'MAXIT', 'ATOL', 'RTOL', 'CONVSPEED'}

The used stopping criterion. 'MAXIT' if the maximum number of iterations *maxit* is reached, 'ATOL' if the objective function value is smaller than *atol*, 'RTOL' if the relative objective function improvement was smaller than *rtol* (i.e. the algorithm converged), 'CONVSPEED' if the objective function improvement is smaller than *convergence_speed*.

rel : float

The relative objective improvement at convergence.

objective : ndarray

The successive evaluations of the objective function at each iteration.

Examples

```
>>> import pyunlocbox
>>> f = pyunlocbox.functions.norm_l2(y=[4, 5, 6, 7])
>>> ret = pyunlocbox.solvers.solve([f], [0, 0, 0, 0], atol=1e-5)
INFO: Dummy objective function added.
INFO: Selected solver : forward_backward
Solution found after 10 iterations :
    objective function f(sol) = 7.460428e-09
    last relative objective improvement : 1.624424e+03
    stopping criterion : ATOL
>>> ret['sol']
array([ 3.99996922,  4.99996153,  5.99995383,  6.99994614])
```

3.3.2 Solver class hierarchy

Solver object interface

class pyunlocbox.solvers.**solver**(*gamma=1*, *post_gamma=None*, *post_sol=None*)

Bases: object

Defines the solver object interface.

This class defines the interface of a solver object intended to be passed to the *pyunlocbox.solvers.solve()* solving function. It is intended to be a base class for standard solvers which will implement the required methods. It can also be instantiated by user code and dynamically modified for rapid testing. This class also defines the generic attributes of all solver objects.

Parameters **gamma** : float

The step size. This parameter is upper bounded by $\frac{1}{\beta}$ where the second convex function (gradient ?) is β Lipschitz continuous. Default is 1.

post_gamma : function

User defined function to post-process the step size. This function is called every iteration and permits the user to alter the solver algorithm. The user may start with a high step size and progressively lower it while the algorithm runs to accelerate the convergence. The function parameters are the following : *gamma* (current step size), *sol* (current problem solution), *objective* (list of successive evaluations of the objective function), *niter* (current iteration number). The function should return a new value for *gamma*. Default is to return an unchanged value.

post_sol : function

User defined function to post-process the problem solution. This function is called every iteration and permits the user to alter the solver algorithm. Same parameter as `post_gamma()`. Default is to return an unchanged value.

algo(*objective*, *niter*)

Call the solver iterative algorithm while allowing the user to alter it. This makes it possible to dynamically change the *gamma* step size while the algorithm is running. See parameters documentation in *pyunlocbox.solvers.solve()* documentation.

post()

Solver specific post-processing. See parameters documentation in *pyunlocbox.solvers.solve()* documentation.

pre (*functions*, *x0*)
Solver specific initialization. See parameters documentation in *pyunlocbox.solvers.solve()* documentation.

Forward-backward proximal splitting algorithm

class pyunlocbox.solvers.**forward_backward**(*method='FISTA'*, *lambda_=1*, **args*, ***kwargs*)
Bases: *pyunlocbox.solvers.solver*

Forward-backward proximal splitting algorithm.

This algorithm solves convex optimization problems composed of the sum of two objective functions.

See generic attributes descriptions of the *pyunlocbox.solvers.solver* base class.

Parameters **method** : {'FISTA', 'ISTA'}, optional

The method used to solve the problem. It can be 'FISTA' or 'ISTA'. Default is 'FISTA'.

lambda_ : float, optional

The update term weight for ISTA. It should be between 0 and 1. Default is 1.

Notes

This algorithm requires one function to implement the *pyunlocbox.functions.func.prox()* method and the other one to implement the *pyunlocbox.functions.func.grad()* method.

Examples

```
>>> from pyunlocbox import functions, solvers
>>> import numpy as np
>>> y = [4, 5, 6, 7]
>>> x0 = np.zeros(len(y))
>>> f1 = functions.norm_l2(y=y)
>>> f2 = functions.dummy()
>>> solver = solvers.forward_backward(method='FISTA', lambda_=1, gamma=1)
>>> ret = solvers.solve([f1, f2], x0, solver, atol=1e-5)
Solution found after 10 iterations :
    objective function f(sol) = 7.460428e-09
    last relative objective improvement : 1.624424e+03
    stopping criterion : ATOL
>>> ret['sol']
array([ 3.99996922,  4.99996153,  5.99995383,  6.99994614])
```

Douglas-Rachford proximal splitting algorithm

class pyunlocbox.solvers.**douglas_rachford**(*lambda_=1*, **args*, ***kwargs*)
Bases: *pyunlocbox.solvers.solver*

Douglas-Rachford proximal splitting algorithm.

This algorithm solves convex optimization problems composed of the sum of two objective functions.

See generic attributes descriptions of the *pyunlocbox.solvers.solver* base class.

Parameters **lambda_** : float, optional

The update term weight. It should be between 0 and 1. Default is 1.

Notes

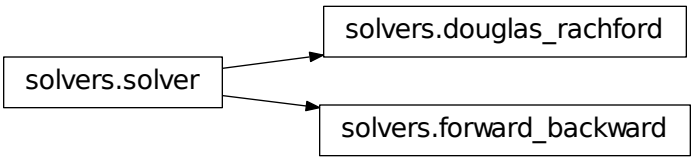
This algorithm requires the two functions to implement the *pyunlocbox.functions.func.prox()* method.

Examples

```
>>> from pyunlocbox import functions, solvers
>>> import numpy as np
>>> y = [4, 5, 6, 7]
>>> x0 = np.zeros(len(y))
>>> f1 = functions.norm_l2(y=y)
>>> f2 = functions.dummy()
>>> solver = solvers.douglas_rachford(lambda_=1, gamma=1)
>>> ret = solvers.solve([f1, f2], x0, solver, atol=1e-5)
Solution found after 8 iterations :
    objective function f(sol) = 2.927052e-06
    last relative objective improvement : 8.000000e+00
    stopping criterion : ATOL
>>> ret['sol']
array([ 3.99939034,  4.99923792,  5.99908551,  6.99893309])
```

This module implements solver objects who minimize an objective function. Call *solve()* to solve your convex optimization problem using your instantiated solver and functions objects. The *solver* base class defines the interface of all solver objects. The specialized solver objects inherit from it and implement the class methods. The following solvers are included :

- *forward_backward*: Forward-backward proximal splitting algorithm.
- *douglas_rachford*: Douglas-Rachford proximal splitting algorithm.



CHAPTER 4

Contributing

Contributions are welcome, and they are greatly appreciated! Every little bit helps, and credit will always be given.

You can contribute in many ways:

4.1 Types of Contributions

4.1.1 Report Bugs

Report bugs at https://github.com/epfl-lts2/pyunlocbox/issues.

If you are reporting a bug, please include:

- Your operating system name and version.
- Any details about your local setup that might be helpful in troubleshooting.
- Detailed steps to reproduce the bug.

4.1.2 Fix Bugs

Look through the GitHub issues for bugs. Anything tagged with "bug" is open to whoever wants to implement it.

4.1.3 Implement Features

Look through the GitHub issues for features. Anything tagged with "feature" is open to whoever wants to implement it.

4.1.4 Write Documentation

pyunlocbox could always use more documentation, whether as part of the official pyunlocbox docs, in docstrings, or even on the web in blog posts, articles, and such.

4.1.5 Submit Feedback

The best way to send feedback is to file an issue at https://github.com/epfl-lts2/pyunlocbox/issues.

If you are proposing a feature:

- Explain in detail how it would work.
- Keep the scope as narrow as possible, to make it easier to implement.
- Remember that this is a volunteer-driven project, and that contributions are welcome :)

4.2 Get Started!

Ready to contribute? Here's how to set up *pyunlocbox* for local development.

1. Fork the *pyunlocbox* repo on GitHub.
2. Clone your fork locally:

```
$ git clone git@github.com:your_name_here/pyunlocbox.git
```

3. Install your local copy into a virtualenv. Assuming you have virtualenvwrapper installed, this is how you set up your fork for local development:

```
$ mkvirtualenv pyunlocbox
$ cd pyunlocbox/
$ python setup.py develop
```

4. Create a branch for local development:

```
$ git checkout -b name-of-your-bugfix-or-feature
```

   Now you can make your changes locally.

5. When you're done making changes, check that your changes pass flake8 and the tests, including testing other Python versions with tox:

```
$ flake8 pyunlocbox tests
$ python setup.py test
$ tox
```

   To get flake8 and tox, just pip install them into your virtualenv.

6. Commit your changes and push your branch to GitHub:

```
$ git add .
$ git commit -m "Your detailed description of your changes."
$ git push origin name-of-your-bugfix-or-feature
```

7. Submit a pull request through the GitHub website.

4.3 Pull Request Guidelines

Before you submit a pull request, check that it meets these guidelines:

1. The pull request should include tests.
2. If the pull request adds functionality, the docs should be updated. Put your new functionality into a function with a docstring, and add the feature to the list in README.rst.
3. The pull request should work for Python 2.6, 2.7, and 3.3, and for PyPy. Check https://travis-ci.org/epfl-lts2/pyunlocbox/pull_requests and make sure that the tests pass for all supported Python versions.

4.4 Tips

To run a subset of tests:

```
$ python -m unittest tests.test_pyunlocbox
```

CHAPTER 5

History

5.1 0.2.0 (2014-08-04)

Second usable version, available on GitHub and released on PyPI. Still experimental.

New features :

- Douglas-Rachford splitting algorithm
- Projection on the L2-ball for tight and non tight frames
- Compressed sensing tutorial using L2-ball, L2-norm and Douglas-Rachford
- Automatic solver selection

Infrastructure :

- Unit tests for all functions and solvers
- Continuous integration testing on Python 2.6, 2.7, 3.2, 3.3 and 3.4

5.2 0.1.0 (2014-06-08)

First usable version, available on GitHub and released on PyPI. Still experimental.

Features :

- Forward-backward splitting algorithm
- L1-norm function (eval and prox)
- L2-norm function (eval, grad and prox)
- Least square problem tutorial using L2-norm and forward-backward
- Compressed sensing tutorial using L1-norm, L2-norm and forward-backward

Infrastructure :

- Sphinx generated documentation using Numpy style docstrings
- Documentation hosted on Read the Docs
- Code hosted on GitHub
- Package hosted on PyPI
- Code checked by flake8

- Docstring and tutorial examples checked by doctest (as a test suite)
- Unit tests for functions module (as a test suite)
- All test suites executed in Python 2.6, 2.7 and 3.2 virtualenvs by tox
- Distributed automatic testing on Travis CI continuous integration platform

CHAPTER 6

Indices and tables

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