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The Python Control Systems Library (python-control) is a Python package that implements basic operations for analysis and design of feedback control systems.

**Features**

- Linear input/output systems in state-space and frequency domain
- Block diagram algebra: serial, parallel, and feedback interconnections
- Time response: initial, step, impulse
- Frequency response: Bode and Nyquist plots
- Control analysis: stability, reachability, observability, stability margins
- Control design: eigenvalue placement, LQR, H2, Hinf
- Model reduction: balanced realizations, Hankel singular values
- Estimator design: linear quadratic estimator (Kalman filter)

**Documentation**
Welcome to the Python Control Systems Toolbox (python-control) User’s Manual. This manual contains information on using the python-control package, including documentation for all functions in the package and examples illustrating their use.

1.1 Overview of the toolbox

The python-control package is a set of python classes and functions that implement common operations for the analysis and design of feedback control systems. The initial goal is to implement all of the functionality required to work through the examples in the textbook Feedback Systems by Astrom and Murray. A MATLAB compatibility module is available that provides many of the common functions corresponding to commands available in the MATLAB Control Systems Toolbox.

1.2 Some differences from MATLAB

The python-control package makes use of NumPy and SciPy. A list of general differences between NumPy and MATLAB can be found here.

In terms of the python-control package more specifically, here are some thing to keep in mind:

- You must include commas in vectors. So [1 2 3] must be [1, 2, 3].
- Functions that return multiple arguments use tuples.
- You cannot use braces for collections; use tuples instead.

1.3 Installation

The python-control package can be installed using pip, conda or the standard distutils/setuptools mechanisms. The package requires numpy and scipy, and the plotting routines require matplotlib. In addition, some routines require the
slycot library in order to implement more advanced features (including some MIMO functionality).

To install using pip:

```bash
pip install slycot  # optional
pip install control
```

Many parts of python-control will work without slycot, but some functionality is limited or absent, and installation of slycot is recommended.

Note: the slycot library only works on some platforms, mostly linux-based. Users should check to insure that slycot is installed correctly by running the command:

```bash
python -c "import slycot"
```

and verifying that no error message appears. It may be necessary to install slycot from source, which requires a working FORTRAN compiler and either the lapack or openplas library. More information on the slycot package can be obtained from the slycot project page.

For users with the Anaconda distribution of Python, the following commands can be used:

```bash
conda install numpy scipy matplotlib  # if not yet installed
conda install -c conda-forge control
```

This installs slycot and python-control from conda-forge, including the openblas package.

Alternatively, to use setuptools, first download the source and unpack it. To install in your home directory, use:

```bash
python setup.py install --user
```

or to install for all users (on Linux or Mac OS):

```bash
python setup.py build
sudo python setup.py install
```

### 1.4 Getting started

There are two different ways to use the package. For the default interface described in Function reference, simply import the control package as follows:

```python
>>> import control
```

If you want to have a MATLAB-like environment, use the MATLAB compatibility module:

```python
>>> from control.matlab import *
```
The python-control library uses a set of standard conventions for the way that different types of standard information used by the library.

## 2.1 LTI system representation

Linear time invariant (LTI) systems are represented in python-control in state space, transfer function, or frequency response data (FRD) form. Most functions in the toolbox will operate on any of these data types and functions for converting between compatible types is provided.

### 2.1.1 State space systems

The `StateSpace` class is used to represent state-space realizations of linear time-invariant (LTI) systems:

\[
\begin{align*}
\frac{dx}{dt} &= Ax + Bu \\
y &=Cx + Du
\end{align*}
\]

where u is the input, y is the output, and x is the state.

To create a state space system, use the `StateSpace` constructor:

```python
sys = StateSpace(A, B, C, D)
```

State space systems can be manipulated using standard arithmetic operations as well as the `feedback()`, `parallel()`, and `series()` function. A full list of functions can be found in Function reference.

### 2.1.2 Transfer functions

The `TransferFunction` class is used to represent input/output transfer functions

\[
G(s) = \frac{\text{num}(s)}{\text{den}(s)} = \frac{a_0 s^m + a_1 s^{m-1} + \cdots + a_m}{b_0 s^n + b_1 s^{n-1} + \cdots + b_n}
\]
where \( n \) is generally greater than or equal to \( m \) (for a proper transfer function).

To create a transfer function, use the `TransferFunction` constructor:

```python
sys = TransferFunction(num, den)
```

Transfer functions can be manipulated using standard arithmetic operations as well as the `feedback()`, `parallel()`, and `series()` function. A full list of functions can be found in `Function reference`.

### 2.1.3 FRD (frequency response data) systems

The `FRD` class is used to represent systems in frequency response data form.

The main data members are `omega` and `fresp`, where `omega` is a 1D array with the frequency points of the response, and `fresp` is a 3D array, with the first dimension corresponding to the output index of the FRD, the second dimension corresponding to the input index, and the 3rd dimension corresponding to the frequency points in `omega`.

FRD systems have a somewhat more limited set of functions that are available, although all of the standard algebraic manipulations can be performed.

### 2.1.4 Discrete time systems

A discrete time system is created by specifying a nonzero ‘timebase’, \( dt \). The timebase argument can be given when a system is constructed:

- \( dt = \text{None} \): no timebase specified (default)
- \( dt = 0 \): continuous time system
- \( dt > 0 \): discrete time system with sampling period ‘\( dt \)’
- \( dt = \text{True} \): discrete time with unspecified sampling period

Only the `StateSpace`, `TransferFunction`, and `InputOutputSystem` classes allow explicit representation of discrete time systems.

Systems must have compatible timebases in order to be combined. A system with timebase `None` can be combined with a system having a specified timebase; the result will have the timebase of the latter system. Similarly, a discrete time system with unspecified sampling time (`\( dt = \text{True} \)`) can be combined with a system having a specified sampling time; the result will be a discrete time system with the sample time of the latter system. For continuous time systems, the `sample_system()` function or the `StateSpace.sample()` and `TransferFunction.sample()` methods can be used to create a discrete time system from a continuous time system. See `Utility functions and conversions`.

### 2.1.5 Conversion between representations

LTI systems can be converted between representations either by calling the constructor for the desired data type using the original system as the sole argument or using the explicit conversion functions `ss2tf()` and `tf2ss()`.

### 2.2 Input/output systems

The `iosys` module contains the `InputOutputSystem` class that represents (possibly nonlinear) input/output systems. The `InputOutputSystem` class is a general class that defines any continuous or discrete time dynamical system. Input/output systems can be simulated and also used to compute equilibrium points and linearizations.
An input/output system is defined as a dynamical system that has a system state as well as inputs and outputs (either
inputs or states can be empty). The dynamics of the system can be in continuous or discrete time. To simulate an
input/output system, use the input_output_response() function:

```python
import python

t, y = input_output_response(io_sys, T, U, X0, params)
```

An input/output system can be linearized around an equilibrium point to obtain a StateSpace linear system. Use
the find_eqpts() function to obtain an equilibrium point and the linearize() function to linearize about that
equilibrium point:

```python
xeq, ueq = find_eqpts(io_sys, X0, U0)
ss_sys = linearize(io_sys, xeq, ueq)
```

Input/output systems can be created from state space LTI systems by using the LinearIOSystem class:

```python
io_sys = LinearIOSystem(ss_sys)
```

Nonlinear input/output systems can be created using the NonlinearIOSystem class, which requires the defini-
tion of an update function (for the right hand side of the differential or different equation) and an output function
(computes the outputs from the state):

```python
io_sys = NonlinearIOSystem(updfcn, outfcn, inputs=M, outputs=P, states=N)
```

More complex input/output systems can be constructed by using the InterconnectedSystem class, which allows
a collection of input/output subsystems to be combined with internal connections between the subsystems and a set of
overall system inputs and outputs that link to the subsystems:

```python
steering = ct.InterconnectedSystem(
    (plant, controller), name='system',
    connections=('controller.e', '-plant.y'),
    inplist=('controller.e'), inputs='r',
    outlist=('plant.y'), outputs='y')
```

Interconnected systems can also be created using block diagram manipulations such as the series(),
parallel(), and feedback() functions. The InputOutputSystem class also supports various algebraic
operations such as * (series interconnection) and + (parallel interconnection).

## 2.3 Time series data

A variety of functions in the library return time series data: sequences of values that change over time. A common set of
conventions is used for returning such data: columns represent different points in time, rows are different components
(e.g., inputs, outputs or states). For return arguments, an array of times is given as the first returned argument, followed
by one or more arrays of variable values. This convention is used throughout the library, for example in the functions
forced_response(), step_response(), impulse_response(), and initial_response().

**Note:** The convention used by python-control is different from the convention used in the scipy.signal library. In
Scipy's convention the meaning of rows and columns is interchanged. Thus, all 2D values must be transposed when
they are used with functions from scipy.signal.

Types:

- **Arguments** can be arrays, matrices, or nested lists.
- **Return values** are arrays (not matrices).
The time vector is either 1D, or 2D with shape (1, n):

\[
T = [[t_1, \quad t_2, \quad t_3, \quad ..., \quad t_n]]
\]

Input, state, and output all follow the same convention. Columns are different points in time, rows are different components. When there is only one row, a 1D object is accepted or returned, which adds convenience for SISO systems:

\[
U = [[u_1(t_1), \quad u_1(t_2), \quad u_1(t_3), \quad ..., \quad u_1(t_n)]
    [u_2(t_1), \quad u_2(t_2), \quad u_2(t_3), \quad ..., \quad u_2(t_n)]
    ...
    ...
    [u_i(t_1), \quad u_i(t_2), \quad u_i(t_3), \quad ..., \quad u_i(t_n)]]
\]

Same `for` `X, Y`

So, `U[:,2]` is the system’s input at the third point in time; and `U[1]` or `U[1,:]` is the sequence of values for the system’s second input.

The initial conditions are either 1D, or 2D with shape (j, 1):

\[
X0 = [[x_1]
      [x_2]
      ...
      ...
      [x_j]]
\]

As all simulation functions return `arrays`, plotting is convenient:

```python
(t, y) = step_response(sys)
plot(t, y)
```

The output of a MIMO system can be plotted like this:

```python
(t, y, x) = forced_response(sys, u, t)
plot(t, y[0], label='y_0')
plot(t, y[1], label='y_1')
```

The convention also works well with the state space form of linear systems. If `D` is the feedthrough matrix of a linear system, and `U` is its input (`matrix` or `array`), then the feedthrough part of the system’s response, can be computed like this:

```python
ft = D * U
```

## 2.4 Package configuration

The python-control library can be customized to allow for different plotting conventions. The currently configurable options allow the units for Bode plots to be set as dB for gain, degrees for phase and Hertz for frequency (MATLAB conventions) or the gain can be given in magnitude units (powers of 10), corresponding to the conventions used in [Feedback Systems (FBS)](https://www.amazon.com/Feedback-Systems-Advanced-Modern-Control/dp/0691117995).

**Variables that can be configured, along with their default values:**

- `bode_dB` (False): Bode plot magnitude plotted in dB (otherwise powers of 10)
- `bode_deg` (True): Bode plot phase plotted in degrees (otherwise radians)
• bode_Hz (False): Bode plot frequency plotted in Hertz (otherwise rad/sec)
• bode_number_of_samples (None): Number of frequency points in Bode plots
• bode_feature_periphery_decade (1.0): How many decades to include in the frequency range on both sides of features (poles, zeros).

Functions that can be used to set standard configurations:

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>use_fbs_defaults()</code></td>
<td>Use Feedback Systems (FBS) compatible settings</td>
</tr>
<tr>
<td><code>use_matlab_defaults()</code></td>
<td>Use MATLAB compatible configuration settings</td>
</tr>
</tbody>
</table>

### 2.4.1 control.use_fbs_defaults

```python
control.use_fbs_defaults()
```
Use Feedback Systems (FBS) compatible settings

**The following conventions are used:**
- Bode plots plot gain in powers of ten, phase in degrees, frequency in Hertz

### 2.4.2 control.use_matlab_defaults

```python
control.use_matlab_defaults()
```
Use MATLAB compatible configuration settings

**The following conventions are used:**
- Bode plots plot gain in dB, phase in degrees, frequency in Hertz
The Python Control Systems Library `control` provides common functions for analyzing and designing feedback control systems.

### 3.1 System creation

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>ss(A, B, C, D[, dt])</code></td>
<td>Create a state space system.</td>
</tr>
<tr>
<td><code>tf(num, den[, dt])</code></td>
<td>Create a transfer function system.</td>
</tr>
<tr>
<td><code>frd(d, w)</code></td>
<td>Construct a frequency response data model</td>
</tr>
<tr>
<td><code>rss([states, outputs, inputs])</code></td>
<td>Create a stable continuous random state space object.</td>
</tr>
<tr>
<td><code>drss([states, outputs, inputs])</code></td>
<td>Create a stable discrete random state space object.</td>
</tr>
</tbody>
</table>

#### 3.1.1 control.ss

`control.ss(A, B, C, D[, dt])`

Create a state space system.

The function accepts either 1, 4 or 5 parameters:

- **ss(sys)** Convert a linear system into space system form. Always creates a new system, even if sys is already a StateSpace object.
- **ss(A, B, C, D)** Create a state space system from the matrices of its state and output equations:
  \[
  \dot{x} = A \cdot x + B \cdot u \\
  y = C \cdot x + D \cdot u
  \]
- **ss(A, B, C, D, dt)** Create a discrete-time state space system from the matrices of its state and output equations:
  \[
  x[k+1] = A \cdot x[k] + B \cdot u[k] \\
  y[k] = C \cdot x[k] + D \cdot u[k]
  \]
The matrices can be given as *array like* data types or strings. Everything that the constructor of `numpy.matrix` accepts is permissible here too.

**Parameters**

- `sys` *(StateSpace or TransferFunction)* – A linear system
- `A` *(array_like or string)* – System matrix
- `B` *(array_like or string)* – Control matrix
- `C` *(array_like or string)* – Output matrix
- `D` *(array_like or string)* – Feed forward matrix
- `dt` *(If present, specifies the sampling period and a discrete time)* – system is created

**Returns out** – The new linear system

**Return type** `StateSpace`

**Raises** `ValueError` – if matrix sizes are not self-consistent

**See also:**

`StateSpace()`, `tf()`, `ss2tf()`, `tf2ss()`

**Examples**

```python
>>> # Create a StateSpace object from four "matrices".
>>> sys1 = ss("1. -2; 3. -4", "5.; 7", "6. 8", "9.")
```

```python
>>> # Convert a TransferFunction to a StateSpace object.
>>> sys_tf = tf([2.], [1., 3])
>>> sys2 = ss(sys_tf)
```

### 3.1.2 control.tf

`control.tf(num, den[, dt])`

Create a transfer function system. Can create MIMO systems.

The function accepts either 1, 2, or 3 parameters:

- `tf(sys)` Convert a linear system into transfer function form. Always creates a new system, even if sys is already a TransferFunction object.

- `tf(num, den)` Create a transfer function system from its numerator and denominator polynomial coefficients.

  If `num` and `den` are 1D array_like objects, the function creates a SISO system.

  To create a MIMO system, `num` and `den` need to be 2D nested lists of array_like objects. (A 3 dimensional data structure in total.) (For details see note below.)

- `tf(num, den, dt)` Create a discrete time transfer function system; `dt` can either be a positive number indicating the sampling time or ‘True’ if no specific timebase is given.

- `tf('s')` or `tf('z')` Create a transfer function representing the differential operator (‘s’) or delay operator (‘z’).
Parameters

- **sys** *(LTI (StateSpace or TransferFunction)) – A linear system*
- **num** *(array_like, or list of list of array_like) – Polynomial coefficients of the numerator*
- **den** *(array_like, or list of list of array_like) – Polynomial coefficients of the denominator*

**Returns** out – The new linear system

**Return type** TransferFunction

**Raises**

- **ValueError** – if *num* and *den* have invalid or unequal dimensions
- **TypeError** – if *num* or *den* are of incorrect type

**See also:**

`TransferFunction()`, `ss()`, `ss2tf()`, `tf2ss()`

**Notes**

num[i][j] contains the polynomial coefficients of the numerator for the transfer function from the (j+1)st input to the (i+1)st output. den[i][j] works the same way.

The list [2, 3, 4] denotes the polynomial $2s^2 + 3s + 4$.

The special forms `tf('s')` and `tf('z')` can be used to create transfer functions for differentiation and unit delays.

**Examples**

```python
>>> # Create a MIMO transfer function object
>>> # The transfer function from the 2nd input to the 1st output is
>>> # (3s + 4) / (6s^2 + 5s + 4).
>>> num = [[[1., 2.], [3., 4.]], [[5., 6.], [7., 8.]]]
>>> den = [[[9., 8., 7.], [6., 5., 4.]], [[3., 2., 1.], [-1., -2., -3.]]]
>>> sys1 = tf(num, den)

>>> # Create a variable 's' to allow algebra operations for SISO systems
>>> s = tf('s')
>>> G = (s + 1)/(s**2 + 2*s + 1)

>>> # Convert a StateSpace to a TransferFunction object.
>>> sys_ss = ss("1. -2; 3. -4", "5.; 7", "6. 8", "9."
>>> sys2 = tf(sys1)
```

### 3.1.3 `control.frd`

`control.frd(d, w)`

Construct a frequency response data model

frd models store the (measured) frequency response of a system.
This function can be called in different ways:

\[ \text{frd(response, freqs)} \] Create an frd model with the given response data, in the form of complex response vector, at matching frequency freqs [in rad/s]

\[ \text{frd(sys, freqs)} \] Convert an LTI system into an frd model with data at frequencies freqs.

**Parameters**
- \( \text{response} (\text{array_like, or list}) \) – complex vector with the system response
- \( \text{freq} (\text{array_like or list}) \) – vector with frequencies
- \( \text{sys} (\text{LTI (StateSpace or TransferFunction)}) \) – A linear system

**Returns** \( \text{sys} \) – New frequency response system

**Return type** \( \text{FRD} \)

See also:
- \( \text{FRD()}, \text{ss()}, \text{tf()} \)

### 3.1.4 control.rss

\[ \text{control.rss (states=1, outputs=1, inputs=1)} \]
Create a stable continuous random state space object.

**Parameters**
- \( \text{states} (\text{integer}) \) – Number of state variables
- \( \text{inputs} (\text{integer}) \) – Number of system inputs
- \( \text{outputs} (\text{integer}) \) – Number of system outputs

**Returns** \( \text{sys} \) – The randomly created linear system

**Return type** \( \text{StateSpace} \)

**Raises** \( \text{ValueError} \) – if any input is not a positive integer

See also:
- \( \text{drss()} \)

**Notes**

If the number of states, inputs, or outputs is not specified, then the missing numbers are assumed to be 1. The poles of the returned system will always have a negative real part.

### 3.1.5 control.drss

\[ \text{control.drss (states=1, outputs=1, inputs=1)} \]
Create a stable discrete random state space object.

**Parameters**
- \( \text{states} (\text{integer}) \) – Number of state variables
- \( \text{inputs} (\text{integer}) \) – Number of system inputs
• **outputs** (integer) – Number of system outputs

Returns sys – The randomly created linear system

Return type **StateSpace**

Raises **ValueError** – if any input is not a positive integer

See also: 

`rss()`

Notes

If the number of states, inputs, or outputs is not specified, then the missing numbers are assumed to be 1. The poles of the returned system will always have a magnitude less than 1.

### 3.2 System interconnections

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>append(sys1, sys2, ..., sysn)</code></td>
<td>Group models by appending their inputs and outputs</td>
</tr>
<tr>
<td><code>connect(sys, Q, inputv, outputv)</code></td>
<td>Index-base interconnection of system</td>
</tr>
<tr>
<td><code>feedback(sys1[, sys2, sign])</code></td>
<td>Feedback interconnection between two I/O systems.</td>
</tr>
<tr>
<td><code>negate(sys)</code></td>
<td>Return the negative of a system.</td>
</tr>
<tr>
<td><code>parallel(sys1, *sysn)</code></td>
<td>Return the parallel connection sys1 + sys2 (+ sys3 + ...)</td>
</tr>
<tr>
<td><code>series(sys1, *sysn)</code></td>
<td>Return the series connection (...</td>
</tr>
</tbody>
</table>

#### 3.2.1 control.append

`control.append(sys1, sys2, ..., sysn)`

Forms an augmented system model, and appends the inputs and outputs together. The system type will be the type of the first system given; if you mix state-space systems and gain matrices, make sure the gain matrices are not first.

**Parameters**

- `sys2, .. sysn (sys1,)` – LTI systems to combine

Returns sys – Combined LTI system, with input/output vectors consisting of all input/output vectors appended

Return type **LTI system**

**Examples**

```python
>>> sys1 = ss("1. -2; 3. -4", "5.; 7", "6. 8", "9.")
>>> sys2 = ss("-1.", "1.", "1.", "0.")
>>> sys = append(sys1, sys2)
```

**Todo:** also implement for transfer function, zpk, etc.
3.2.2 control.connect

control.connect(sys, Q, inputv, outputv)
Index-base interconnection of system

The system sys is a system typically constructed with append, with multiple inputs and outputs. The inputs and outputs are connected according to the interconnection matrix Q, and then the final inputs and outputs are trimmed according to the inputs and outputs listed in inputv and outputv.

Note: to have this work, inputs and outputs start counting at 1!!!!

Parameters

• sys (StateSpace Transferfunction) – System to be connected
• Q (2d array) – Interconnection matrix. First column gives the input to be connected, second column gives the output to be fed into this input. Negative values for the second column mean the feedback is negative, 0 means no connection is made
• inputv (1d array) – list of final external inputs
• outputv (1d array) – list of final external outputs

Returns sys – Connected and trimmed LTI system

Return type LTI system

Examples

```python
>>> sys1 = ss("1. -2; 3. -4", "5.; 7", "6, 8", "9."
>>> sys2 = ss("-1.", "1.", "1.", "0.")
>>> sys = append(sys1, sys2)
>>> Q = sp.mat([ [ 1, 2], [2, -1] ])
# basically feedback, output 2 in 1
>>> sysc = connect(sys, Q, [2], [1, 2])
```

3.2.3 control.feedback

control.feedback(sys1, sys2=1, sign=-1)
Feedback interconnection between two I/O systems.

Parameters

• sys1 (scalar, StateSpace, TransferFunction, FRD) – The primary plant.
• sys2 (scalar, StateSpace, TransferFunction, FRD) – The feedback plant (often a feedback controller).
• sign (scalar) – The sign of feedback. sign = -1 indicates negative feedback, and sign = 1 indicates positive feedback. sign is an optional argument; it assumes a value of -1 if not specified.

Returns out

Return type StateSpace or TransferFunction

Raises

• ValueError – if sys1 does not have as many inputs as sys2 has outputs, or if sys2 does not have as many inputs as sys1 has outputs
• `NotImplementedError` – if an attempt is made to perform a feedback on a MIMO TransferFunction object

See also:

`series()`, `parallel()`

Notes

This function is a wrapper for the feedback function in the StateSpace and TransferFunction classes. It calls TransferFunction.feedback if `sys1` is a TransferFunction object, and StateSpace.feedback if `sys1` is a StateSpace object. If `sys1` is a scalar, then it is converted to `sys2`'s type, and the corresponding feedback function is used. If `sys1` and `sys2` are both scalars, then TransferFunction.feedback is used.

3.2.4 `control.negate`

`control.negate(sys)`

Return the negative of a system.

Parameters `sys` (StateSpace, TransferFunction or FRD) –

Returns out

Return type StateSpace or TransferFunction

Notes

This function is a wrapper for the `__neg__` function in the StateSpace and TransferFunction classes. The output type is the same as the input type.

If both systems have a defined timebase (dt = 0 for continuous time, dt > 0 for discrete time), then the timebase for both systems must match. If only one of the system has a timebase, the return timebase will be set to match it.

Examples

```python
>>> sys2 = negate(sys1)  # Same as sys2 = -sys1.
```

3.2.5 `control.parallel`

`control.parallel(sys1, *sysn)`

Return the parallel connection `sys1 + sys2 (+ sys3 + ...)`

Parameters

- `sys1` (scalar, StateSpace, TransferFunction, or FRD) –
- `*sysn` (other scalars, StateSpaces, TransferFunctions, or FRDs) –

Returns out

Return type scalar, StateSpace, or TransferFunction

Raises ValueError – if `sys1` and `sys2` do not have the same numbers of inputs and outputs
See also:

series(), feedback()

Notes

This function is a wrapper for the __add__ function in the StateSpace and TransferFunction classes. The output type is usually the type of sys1. If sys1 is a scalar, then the output type is the type of sys2.

If both systems have a defined timebase (dt = 0 for continuous time, dt > 0 for discrete time), then the timebase for both systems must match. If only one of the system has a timebase, the return timebase will be set to match it.

Examples

```python
>>> sys3 = parallel(sys1, sys2)  # Same as sys3 = sys1 + sys2

>>> sys5 = parallel(sys1, sys2, sys3, sys4)  # More systems
```

3.2.6 control.series

control.series(sys1, *sysn)

Return the series connection (… * sys3 *) sys2 * sys1

Parameters

- sys1 (scalar, StateSpace, TransferFunction, or FRD)-
- sysn (other scalars, StateSpaces, TransferFunctions, or FRDs)-

Returns out

Return type scalar, StateSpace, or TransferFunction

Raises ValueError – if sys2.inputs does not equal sys1.outputs if sys1.dt is not compatible with sys2.dt

See also:

parallel(), feedback()

Notes

This function is a wrapper for the __mul__ function in the StateSpace and TransferFunction classes. The output type is usually the type of sys2. If sys2 is a scalar, then the output type is the type of sys1.

If both systems have a defined timebase (dt = 0 for continuous time, dt > 0 for discrete time), then the timebase for both systems must match. If only one of the system has a timebase, the return timebase will be set to match it.

Examples

```python
>>> sys3 = series(sys1, sys2)  # Same as sys3 = sys2 * sys1
```
```python
>>> sys5 = series(sys1, sys2, sys3, sys4)  # More systems
```

## 3.3 Frequency domain plotting

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### 3.3.1 control.bode_plot

```python
control.bode_plot(syslist, omega=None, dB=None, Hz=None, deg=None, Plot=True, omega_limits=None, omega_num=None, margins=None, *args, **kwargs)
```

Bode plot for a system

Plots a Bode plot for the system over a (optional) frequency range.

**Parameters**

- `syslist` *(linsys)* – List of linear input/output systems (single system is OK)
- `omega` *(list)* – List of frequencies in rad/sec to be used for frequency response
- `dB` *(boolean)* – If True, plot result in dB
- `Hz` *(boolean)* – If True, plot frequency in Hz (omega must be provided in rad/sec)
- `deg` *(boolean)* – If True, plot phase in degrees (else radians)
- `Plot` *(boolean)* – If True, plot magnitude and phase
- `omega_limits` *(tuple, list, . . of two values)* – Limits of the to generate frequency vector. If Hz=True the limits are in Hz otherwise in rad/s.
- `omega_num` *(int)* – number of samples
- `margins` *(boolean)* – If True, plot gain and phase margin
- `**kwargs` *(*args, **kwargs)* – Additional options to matplotlib (color, linestyle, etc)

**Returns**

- `mag` *(array (list if len(syslist) > 1))* – magnitude
- `phase` *(array (list if len(syslist) > 1))* – phase in radians
- `omega` *(array (list if len(syslist) > 1))* – frequency in rad/sec

**Notes**

1. Alternatively, you may use the lower-level method `(mag, phase, freq) = sys.freqresp(freq)` to generate the frequency response for a system, but it returns a MIMO response.
2. If a discrete time model is given, the frequency response is plotted along the upper branch of the unit circle, using the mapping \( z = \exp(j \omega dt) \) where \( \omega \) ranges from 0 to \( \pi/dt \) and \( dt \) is the discrete timebase. If not timebase is specified (\( dt = True \), \( dt \) is set to 1.)
Examples

```python
>>> sys = ss("1. -2; 3. -4", "5.; 7", "6. 8", "9.")
>>> mag, phase, omega = bode(sys)
```

### 3.3.2 control.nyquist_plot

The function `control.nyquist_plot(syslist, omega=None, Plot=True, color=None, labelFreq=0, **kwargs)` generates a Nyquist plot for a system over a specified frequency range. The function takes the following parameters:

- `syslist` (list of LTI) – A list of linear time-invariant systems (single system is also accepted).
- `omega` (freq_range) – A range of frequencies (list or bounds) in radians per second.
- `Plot` (boolean) – If True, plot the magnitude.
- `color` (string) – Used to specify the color of the plot.
- `labelFreq` (int) – Label every `n`th frequency on the plot.
- `**kwargs` (args, **kwargs) – Additional options to `matplotlib` (color, linestyle, etc).

The function returns the real part, imaginary part, and the frequency array.

Examples

```python
>>> sys = ss("1. -2; 3. -4", "5.; 7", "6. 8", "9.")
>>> real, imag, freq = nyquist_plot(sys)
```

### 3.3.3 control.gangof4_plot

The function `control.gangof4_plot(P, C, omega=None)` plots the “Gang of 4” transfer functions for a system. The function takes the following parameters:

- `P` – Process system.
- `C` – Controller system.
- `omega` (array) – A range of frequencies (list or bounds) in radians per second.

The function generates a 2x2 plot showing the “Gang of 4” sensitivity functions [T, PS; CS, S].

Parameters

- `C` (P) – Linear input/output systems (process and control).
- `omega` (array) – Range of frequencies (list or bounds) in radians per second.

Returns

- `None`

3.3.4 control.nichols_plot

control.nichols_plot(sys_list, omega=None, grid=True)
Nichols plot for a system

Plots a Nichols plot for the system over a (optional) frequency range.

Parameters

• sys_list (list of LTI, or LTI) – List of linear input/output systems (single system is OK)
• omega (array_like) – Range of frequencies (list or bounds) in rad/sec
• grid (boolean, optional) – True if the plot should include a Nichols-chart grid. Default is True.

Returns

Return type None

Note: For plotting commands that create multiple axes on the same plot, the individual axes can be retrieved using the axes label (retrieved using the get_label method for the matplotlib axes object). The following labels are currently defined:

• Bode plots: control-bode-magnitude, control-bode-phase
• Gang of 4 plots: control-gangof4-s, control-gangof4-cs, control-gangof4-ps, control-gangof4-t

3.4 Time domain simulation

forced_response(sys[, T, U, X0, transpose, ...]) Simulate the output of a linear system.

impulse_response(sys[, T, X0, input, ...]) Impulse response of a linear system

initial_response(sys[, T, X0, input, ...]) Initial condition response of a linear system

input_output_response(sys, T[, U, X0, ...]) Compute the output response of a system to a given input.

step_response(sys[, T, X0, input, output, ...]) Step response of a linear system

phase_plot(odefun[, X, Y, scale, X0, T, ...]) Phase plot for 2D dynamical systems

3.4.1 control.forced_response

control.forced_response(sys, T=None, U=0.0, X0=0.0, transpose=False, interpolate=False, squeeze=True)
Simulate the output of a linear system.

As a convenience for parameters U, X0: Numbers (scalars) are converted to constant arrays with the correct shape. The correct shape is inferred from arguments sys and T.

For information on the shape of parameters U, T, X0 and return values T, yout, xout, see Time series data.

Parameters

• sys (LTI (StateSpace, or TransferFunction)) – LTI system to simulate
• T (array-like) – Time steps at which the input is defined; values must be evenly spaced.
• U (array-like or number, optional) – Input array giving input at each time T (default = 0).
If $U$ is None or 0, a special algorithm is used. This special algorithm is faster than the general algorithm, which is used otherwise.

- **X0** (array-like or number, optional) – Initial condition (default = 0).
- **transpose** (bool, optional (default=False)) – If True, transpose all input and output arrays (for backward compatibility with MATLAB and scipy.signal.lsim).
- **interpolate** (bool, optional (default=False)) – If True and system is a discrete time system, the input will be interpolated between the given time steps and the output will be given at system sampling rate. Otherwise, only return the output at the times given in $T$. No effect on continuous time simulations (default = False).
- **squeeze** (bool, optional (default=True)) – If True, remove single-dimensional entries from the shape of the output. For single output systems, this converts the output response to a 1D array.

**Returns**

- **T** (array) – Time values of the output.
- **yout** (array) – Response of the system.
- **xout** (array) – Time evolution of the state vector.

**See also:**

`step_response()`, `initial_response()`, `impulse_response()`

**Examples**

```python
>>> T, yout, xout = forced_response(sys, T, u, X0)
```

See *Time series data*.

### 3.4.2 control.impulse_response

**control.impulse_response** (sys, T=None, X0=0.0, input=0, output=None, transpose=False, return_x=False, squeeze=True)

Impulse response of a linear system

If the system has multiple inputs or outputs (MIMO), one input has to be selected for the simulation. Optionally, one output may be selected. The parameters `input` and `output` do this. All other inputs are set to 0, all other outputs are ignored.

For information on the shape of parameters $T$, $X0$ and return values $T$, $yout$, see *Time series data*.

**Parameters**

- **sys** (StateSpace, TransferFunction) – LTI system to simulate
- **T** (array-like object, optional) – Time vector (argument is autocomputed if not given)
- **X0** (array-like object or number, optional) – Initial condition (default = 0)

Numbers are converted to constant arrays with the correct shape.
- **input** (int) – Index of the input that will be used in this simulation.
• **output** (*int*) – Index of the output that will be used in this simulation. Set to None to not trim outputs

• **transpose** (*bool*) – If True, transpose all input and output arrays (for backward compatibility with MATLAB and scipy.signal.lsim)

• **return_x** (*bool*) – If True, return the state vector (default = False).

• **squeeze** (*bool, optional (default=True)*) – If True, remove single-dimensional entries from the shape of the output. For single output systems, this converts the output response to a 1D array.

**Returns**

• **T** (*array*) – Time values of the output

• **yout** (*array*) – Response of the system

• **xout** (*array*) – Individual response of each x variable

See also:

`forced_response()`, `initial_response()`, `step_response()`

**Examples**

```python
>>> T, yout = impulse_response(sys, T, X0)
```

### 3.4.3 control.initial_response

**control.initial_response** *(sys, T=None, X0=0.0, input=0, output=None, transpose=False, return_x=False, squeeze=True)*

Initial condition response of a linear system

If the system has multiple outputs (MIMO), optionally, one output may be selected. If no selection is made for the output, all outputs are given.

For information on the shape of parameters T, X0 and return values T, yout, see Time series data.

**Parameters**

• **sys** (*StateSpace, or TransferFunction*) – LTI system to simulate

• **T** (*array-like object, optional) – Time vector (argument is autocomputed if not given)

• **X0** (*array-like object or number, optional) – Initial condition (default = 0)

  Numbers are converted to constant arrays with the correct shape.

• **input** (*int*) – Ignored, has no meaning in initial condition calculation. Parameter ensures compatibility with step_response and impulse_response

• **output** (*int*) – Index of the output that will be used in this simulation. Set to None to not trim outputs

• **transpose** (*bool*) – If True, transpose all input and output arrays (for backward compatibility with MATLAB and scipy.signal.lsim)

• **return_x** (*bool*) – If True, return the state vector (default = False).
• **squeeze** *(bool, optional (default=True)) –* If True, remove single-dimensional entries from the shape of the output. For single output systems, this converts the output response to a 1D array.

Returns

• **T** *(array) –* Time values of the output

• **yout** *(array) –* Response of the system

• **xout** *(array) –* Individual response of each x variable

See also:

```
forced_response(), impulse_response(), step_response()
```

### Examples

```python
>>> T, yout = initial_response(sys, T, X0)
```

#### 3.4.4 control.input_output_response

```python
control.input_output_response(sys, T, U=0.0, X0=0, params={}, method='RK45', return_x=False, squeeze=True)
```

Compute the output response of a system to a given input.

Simulate a dynamical system with a given input and return its output and state values.

**Parameters**

• **sys** *(InputOutputSystem) –* Input/output system to simulate.

• **T** *(array-like) –* Time steps at which the input is defined; values must be evenly spaced.

• **U** *(array-like or number, optional) –* Input array giving input at each time T (default = 0).

• **X0** *(array-like or number, optional) –* Initial condition (default = 0).

• **return_x** *(bool, optional) –* If True, return the values of the state at each time (default = False).

• **squeeze** *(bool, optional) –* If True (default), squeeze unused dimensions out of the output response. In particular, for a single output system, return a vector of shape (nsteps) instead of (nsteps, 1).

**Returns**

• **T** *(array) –* Time values of the output.

• **yout** *(array) –* Response of the system.

• **xout** *(array) –* Time evolution of the state vector (if return_x=True)

**Raises**

• **TypeError** – If the system is not an input/output system.

• **ValueError** – If time step does not match sampling time (for discrete time systems)
### 3.4.5 control.step_response

**control.step_response** *(sys, T=None, X0=0.0, input=None, output=None, transpose=False, return_x=False, squeeze=True)*

Step response of a linear system

If the system has multiple inputs or outputs (MIMO), one input has to be selected for the simulation. Optionally, one output may be selected. The parameters *input* and *output* do this. All other inputs are set to 0, all other outputs are ignored.

For information on the shape of parameters *T, X0* and return values *T, yout*, see *Time series data*.

**Parameters**

- **sys** *(StateSpace, or TransferFunction)* – LTI system to simulate
- **T** *(array-like object, optional)* – Time vector (argument is autocomputed if not given)
- **X0** *(array-like or number, optional)* – Initial condition (default = 0)
  Numbers are converted to constant arrays with the correct shape.
- **input** *(int)* – Index of the input that will be used in this simulation.
- **output** *(int)* – Index of the output that will be used in this simulation. Set to None to not trim outputs
- **transpose** *(bool)* – If True, transpose all input and output arrays (for backward compatibility with MATLAB and scipy.signal.lsim)
- **return_x** *(bool)* – If True, return the state vector (default = False).
- **squeeze** *(bool, optional (default=True))* – If True, remove single-dimensional entries from the shape of the output. For single output systems, this converts the output response to a 1D array.

**Returns**

- **T** *(array)* – Time values of the output
- **yout** *(array)* – Response of the system
- **xout** *(array)* – Individual response of each x variable

**See also:**

*forced_response(), initial_response(), impulse_response ()*

**Examples**

```python
>>> T, yout = step_response(sys, T, X0)
```

### 3.4.6 control.phase_plot

**control.phase_plot** *(odefun, X=None, Y=None, scale=1, X0=None, T=None, lingrid=None, lin-time=None, logtime=None, timepts=None, parms=(), verbose=True)*

Phase plot for 2D dynamical systems

Produces a vector field or stream line plot for a planar system.
Call signatures:  
phase_plot(func, X, Y, ...) - display vector field on meshgrid  
phase_plot(func, X, Y, scale, ...) - scale arrows  
phase_plot(func, X0=(...), T=Tmax, ...) - display stream lines  
phase_plot(func, X, Y, X0=[...], T=Tmax, ...) - plot both  
phase_plot(func, X0=[...], T=Tmax, lintime=N, ...) - stream lines with arrows

Parameters

• func(callable(x, t, ..)) – Computes the time derivative of y (compatible with odeint). The function should be the same for as used for scipy.integrate. Namely, it should be a function of the form dx/dt = F(x, t) that accepts a state x of dimension 2 and returns a derivative dx/dt of dimension 2.

• Y(X) – Two 3-element sequences specifying x and y coordinates of a grid. These arguments are passed to linspace and meshgrid to generate the points at which the vector field is plotted. If absent (or None), the vector field is not plotted.

• scale(float, optional) – Scale size of arrows; default = 1

• X0(ndarray of initial conditions, optional) – List of initial conditions from which streamlines are plotted. Each initial condition should be a pair of numbers.

• T(array-like or number, optional) – Length of time to run simulations that generate streamlines. If a single number, the same simulation time is used for all initial conditions. Otherwise, should be a list of length len(X0) that gives the simulation time for each initial condition. Default value = 50.

• = N or (N, M) (lingrid) – If X0 is given and X, Y are missing, a grid of arrows is produced using the limits of the initial conditions, with N grid points in each dimension or M grid points in y.

• = N(lintime) – Draw N arrows using equally space time points

• = (N, lambda) (logtime) – Draw N arrows using exponential time constant lambda

• = [t1, t2, ..] (timepts) – Draw arrows at the given list times

• parms(tuple, optional) – List of parameters to pass to vector field: func(x, t, *parms)

See also:

box_grid() construct box-shaped grid of initial conditions

Examples

3.5 Block diagram algebra

**series(sys1,*sysn)** Return the series connection (...)

**parallel(sys1,*sysn)** Return the parallel connection sys1 + sys2 (+ sys3 + ...)

**feedback(sys1[, sys2, sign])** Feedback interconnection between two I/O systems.

**negate(sys)** Return the negative of a system.

3.6 Control system analysis
### Python Control Library Documentation, Release dev

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#### 3.6.1 control.dcgain

**control.dcgain(sys)**

Return the zero-frequency (or DC) gain of the given system

**Returns**
- `gain` – The zero-frequency gain, or np.nan if the system has a pole at the origin

**Return type** `ndarray`

#### 3.6.2 control.evalfr

**control.evalfr(sys, x)**

Evaluate the transfer function of an LTI system for a single complex number x.

To evaluate at a frequency, enter x = omega*j, where omega is the frequency in radians

**Parameters**
- `sys` (*StateSpace* or *TransferFunction*) – Linear system
- `x` (*scalar*) – Complex number

**Returns**
- `fresp` – Complex number

**Return type** `ndarray`

**See also:**
- `freqresp()`, `bode()`

**Notes**

This function is a wrapper for StateSpace.evalfr and TransferFunction.evalfr.
Examples

```python
>>> sys = ss("1. -2; 3. -4", "5.; 7", "6. 8", "9."
>>> evalfr(sys, 1j)
array([[ 44.8-21.4j]])
>>> # This is the transfer function matrix evaluated at s = i.
```

Todo: Add example with MIMO system

3.6.3 control.freqresp

ccontrol.freqresp(sys, omega)

Frequency response of an LTI system at multiple angular frequencies.

Parameters

- `sys` ([StateSpace](#) or [TransferFunction](#)) – Linear system
- `omega` (array_like) – List of frequencies

Returns

- `mag` (ndarray)
- `phase` (ndarray)
- `omega` (list, tuple, or ndarray)

See also:

evalfr(), bode()

Notes

This function is a wrapper for StateSpace.freqresp and TransferFunction.freqresp. The output omega is a sorted version of the input omega.

Examples

```python
>>> sys = ss("1. -2; 3. -4", "5.; 7", "6. 8", "9."
>>> mag, phase, omega = freqresp(sys, [0.1, 1., 10.])
>>> mag
array([[ 58.8576682 , 49.64876635, 13.40825927]])
>>> phase
array([[ -0.05408304, -0.44563154, -0.66837155]])
>>> # This is the magnitude of the frequency response from the
>>> # 2nd input to the 1st output, and the
>>> # phase (in radians) of the
>>> # frequency response from the 1st input to the 2nd output, for
>>> # s = 0.1i, i, 10i.
```

Todo: Add example with MIMO system

#>>> sys = rss(3, 2, 2) #>>> mag, phase, omega = freqresp(sys, [0.1, 1., 10.]) #>>> mag[0, 1, :] #array([55.43747231, 42.47766549, 1.97225895]) #>>> phase[1, 0, :] #array([-0.12611087, -1.14294316, 2.5764547]) #>>> # This is the magnitude of the frequency response from the 2nd
#>>> # input to the 1st output, and the
#>>> # phase (in radians) of the
#>>> # frequency response from the 1st input to the 2nd output, for
#>>> # s = 0.1i, i, 10i.
### 3.6.4 control.margin

`control.margin(sysdata)`

Calculate gain and phase margins and associated crossover frequencies

**Parameters**

- `sysdata` *(LTI system or (mag, phase, omega) sequence)*
  - `sys` [StateSpace or TransferFunction] Linear SISO system
  - `mag`, `phase`, `omega` [sequence of array_like] Input magnitude, phase (in deg.), and frequencies (rad/sec) from bode frequency response data

**Returns**

- `gm` *(float)* – Gain margin
- `pm` *(float)* – Phase margin (in degrees)
- `wg` *(float)* – Frequency for gain margin (at phase crossover, phase = -180 degrees)
- `wp` *(float)* – Frequency for phase margin (at gain crossover, gain = 1)
- *Margins are calculated for a SISO open-loop system.*
- *If there is more than one gain crossover, the one at the smallest margin (deviation from gain = 1), in absolute sense, is returned. Likewise the smallest phase margin (in absolute sense) is returned.*

**Examples**

```python
>>> sys = tf(1, [1, 2, 1, 0])
>>> gm, pm, wg, wp = margin(sys)
```

### 3.6.5 control.stability_margins

`control.stability_margins(sysdata, returnall=False, epsw=0.0)`

Calculate stability margins and associated crossover frequencies.

**Parameters**

- `sysdata` *(LTI system or (mag, phase, omega) sequence)*
  - `sys` [LTI system] Linear SISO system
  - `mag`, `phase`, `omega` [sequence of array_like] Arrays of magnitudes (absolute values, not dB), phases (degrees), and corresponding frequencies. Crossover frequencies returned are in the same units as those in `omega` (e.g., rad/sec or Hz).
- `returnall` *(bool, optional)* – If true, return all margins found. If False (default), return only the minimum stability margins. For frequency data or FRD systems, only margins in the given frequency region can be found and returned.
- `epsw` *(float, optional)* – Frequencies below this value (default 0.0) are considered static gain, and not returned as margin.

**Returns**

- `gm` *(float or array_like)* – Gain margin
• **pm** (*float or array_like*) – Phase margin
• **sm** (*float or array_like*) – Stability margin, the minimum distance from the Nyquist plot to -1
• **wg** (*float or array_like*) – Frequency for gain margin (at phase crossover, phase = -180 degrees)
• **wp** (*float or array_like*) – Frequency for phase margin (at gain crossover, gain = 1)
• **ws** (*float or array_like*) – Frequency for stability margin (complex gain closest to -1)

### 3.6.6 control.phase_crossover_frequencies

`control.phase_crossover_frequencies(sys)`  
Compute frequencies and gains at intersections with real axis in Nyquist plot.

**Call as:** `omega, gain = phase_crossover_frequencies()`  

**Returns**  
- **omega** (*1d array of (non-negative) frequencies where Nyquist plot intersects the real axis*)  
- **gain** (*1d array of corresponding gains*)

**Examples**

```python
>>> tf = TransferFunction([1], [1, 2, 3, 4])
>>> PhaseCrossoverFrequencies(tf)
(array([ 1.73205081,  0.       ]), array([-0.5 ,  0.25]))
```

### 3.6.7 control.pole

`control.pole(sys)`  
Compute system poles.

**Parameters**  
- **sys** (*StateSpace or TransferFunction*) – Linear system  

**Returns**  
- **poles** (*Array that contains the system’s poles.*)  

**Return type**  
- *ndarray*

**Raises**  
- *NotImplementedError* – when called on a TransferFunction object

**See also:**
- `zero()`, `TransferFunction.pole()`, `StateSpace.pole()`

### 3.6.8 control.zero

`control.zero(sys)`  
Compute system zeros.

**Parameters**  
- **sys** (*StateSpace or TransferFunction*) – Linear system

**Returns**  
- **zeros** (*Array that contains the system’s zeros.*)
Return type  ndarray

Raises  NotImplementedError – when called on a MIMO system

See also:

pole(), StateSpace.zero(), TransferFunction.zero()

3.6.9 control.pzmap

control.pzmap(sys, Plot=True, grid=False, title='Pole Zero Map')

Plot a pole/zero map for a linear system.

Parameters

• sys (LTI (StateSpace or TransferFunction)) – Linear system for which poles and zeros are computed.

• Plot (bool) – If True a graph is generated with Matplotlib, otherwise the poles and zeros are only computed and returned.

• grid (boolean (default = False)) – If True plot omega-damping grid.

Returns

• pole (array) – The systems poles

• zeros (array) – The system’s zeros.

3.6.10 control.root_locus

control.root_locus(sys, kvect=None, xlim=None, ylim=None, plotstr='C0', Plot=True, PrintGain=True, grid=False, **kwargs)

Root locus plot

Calculate the root locus by finding the roots of 1+k*TF(s) where TF is self.num(s)/self.den(s) and each k is an element of kvect.

Parameters

• sys (LTI object) – Linear input/output systems (SISO only, for now).

• kvect (list or ndarray, optional) – List of gains to use in computing diagram.

• xlim (tuple or list, optional) – Set limits of x axis, normally with tuple (see matplotlib.axes).

• ylim (tuple or list, optional) – Set limits of y axis, normally with tuple (see matplotlib.axes).

• Plot (boolean, optional) – If True (default), plot root locus diagram.

• PrintGain (bool) – If True (default), report mouse clicks when close to the root locus branches, calculate gain, damping and print.

• grid (bool) – If True plot omega-damping grid. Default is False.

Returns

• rlist (ndarray) – Computed root locations, given as a 2D array

• klist (ndarray or list) – Gains used. Same as klist keyword argument if provided.
3.6.11 control.sisotool

control.sisotool(sys, kvect=None, xlim_rlocus=None, ylim_rlocus=None, plotstr_rlocus='C0', rlocus_grid=False, omega=None, dB=None, Hz=None, deg=None, omega_limits=None, omega_num=None, margins_bode=True, tvect=None)

Sisotool style collection of plots inspired by MATLAB’s sisotool. The left two plots contain the bode magnitude and phase diagrams. The top right plot is a clickable root locus plot, clicking on the root locus will change the gain of the system. The bottom left plot shows a closed loop time response.

Parameters

- **sys** *(LTI object)* – Linear input/output systems (SISO only)
- **kvect** *(list or ndarray, optional)* – List of gains to use for plotting root locus
- **xlim_rlocus** *(tuple or list, optional)* – control of x-axis range, normally with tuple (see matplotlib.axes)
- **ylim_rlocus** *(tuple or list, optional)* – control of y-axis range
- **plotstr_rlocus** *(Additional options to matplotlib)* – plotting style for the root locus plot(color, linestyle, etc)
- **rlocus_grid** *(boolean (default = False))* – If True plot s-plane grid.
- **omega** *(freq_range)* – Range of frequencies in rad/sec for the bode plot
- **dB** *(boolean)* – If True, plot result in dB for the bode plot
- **Hz** *(boolean)* – If True, plot frequency in Hz for the bode plot (omega must be provided in rad/sec)
- **deg** *(boolean)* – If True, plot phase in degrees for the bode plot (else radians)
- **omega_limits** *(tuple, list, .. of two values)* – Limits of the to generate frequency vector. If Hz=True the limits are in Hz otherwise in rad/s.
- **omega_num** *(int)* – number of samples
- **margins_bode** *(boolean)* – If True, plot gain and phase margin in the bode plot
- **tvect** *(list or ndarray, optional)* – List of timesteps to use for closed loop step response

Examples

```python
>>> sys = tf([1000], [1,25,100,0])
>>> sisotool(sys)
```

3.7 Matrix computations

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>care</strong> <em>(A, B, Q[, R, S, E, stabilizing])</em></td>
<td><em>(X,L,G) = care(A,B,Q,R=None)</em> solves the continuous-time algebraic Riccati equation</td>
</tr>
<tr>
<td><strong>dare</strong> <em>(A, B, Q, R[, S, E, stabilizing])</em></td>
<td><em>(X,L,G) = dare(A,B,Q,R)</em> solves the discrete-time algebraic Riccati equation</td>
</tr>
<tr>
<td><strong>lyap</strong> <em>(A, Q[, C, E])</em></td>
<td><em>X = lyap(A,Q)</em> solves the continuous-time Lyapunov equation</td>
</tr>
</tbody>
</table>

Continued on next page
### 3.7.1 control.care

**control.care** *(A, B, Q, R=None, S=None, E=None, stabilizing=True)*

(X, L, G) = care(A, B, Q, R=None) solves the continuous-time algebraic Riccati equation

\[
A^T X A - X A^T - X B R^{-1} B^T X + Q = 0
\]

where A and Q are square matrices of the same dimension. Further, Q and R are a symmetric matrices. If R is None, it is set to the identity matrix. The function returns the solution X, the gain matrix \( G = B^T X \) and the closed loop eigenvalues L, i.e., the eigenvalues of \( A - B G \).

(X, L, G) = care(A, B, Q, R, S, E) solves the generalized continuous-time algebraic Riccati equation

\[
A^T X E + E^T X A - (E^T X B + S) R^{-1} (B^T X E + S^T) + Q = 0
\]

where A, Q and E are square matrices of the same dimension. Further, Q and R are symmetric matrices. If R is None, it is set to the identity matrix. The function returns the solution X, the gain matrix \( G = R^{-1} (B^T X E + S^T) \) and the closed loop eigenvalues L, i.e., the eigenvalues of \( A - B G , E \).

### 3.7.2 control.dare

**control.dare** *(A, B, Q, R, S=None, E=None, stabilizing=True)*

(X, L, G) = dare(A, B, Q, R) solves the discrete-time algebraic Riccati equation

\[
A^T X A - X A^T - X B R^{-1} B^T X + Q = 0
\]

where A and Q are square matrices of the same dimension. Further, Q and R are symmetric matrices. If R is None, it is set to the identity matrix. The function returns the solution X, the gain matrix \( G = (B^T X B + R)^{-1} B^T X A \) and the closed loop eigenvalues L, i.e., the eigenvalues of \( A - B G \).

(X, L, G) = dare(A, B, Q, R, S, E) solves the generalized discrete-time algebraic Riccati equation

\[
A^T X A - E^T X E - (A^T X B + S) (B^T X B + R)^{-1} (B^T X A + S^T) + Q = 0
\]

where A, Q and E are square matrices of the same dimension. Further, Q and R are symmetric matrices. The function returns the solution X, the gain matrix \( G = (B^T X B + R)^{-1} (B^T X A + S^T) \) and the closed loop eigenvalues L, i.e., the eigenvalues of \( A - B G , E \).

### 3.7.3 control.lyap

**control.lyap** *(A, Q, C=None, E=None)*

X = lyap(A, Q) solves the continuous-time Lyapunov equation

\[
AX + X A^T + Q = 0
\]

where A and Q are square matrices of the same dimension. Further, Q must be symmetric.

X = lyap(A, Q, C) solves the Sylvester equation

\[
AX + X Q + C = 0
\]

where A and Q are square matrices.

X = lyap(A, Q, None, E) solves the generalized continuous-time Lyapunov equation
\[ AXE^T + EXAT + Q = 0 \]

where \( Q \) is a symmetric matrix and \( A, Q \) and \( E \) are square matrices of the same dimension.

### 3.7.4 control.dlyap

def dlyap(A, Q, C=None, E=None):
    # dlyap(A,Q) solves the discrete-time Lyapunov equation
    # \[ AXA^T + X + Q = 0 \]
    # where \( A \) and \( Q \) are square matrices of the same dimension. Further \( Q \) must be symmetric.
    # dlyap(A,Q,C) solves the Sylvester equation
    # \[ AXQ^T + X + C = 0 \]
    # where \( A \) and \( Q \) are square matrices.
    # dlyap(A,Q,None,E) solves the generalized discrete-time Lyapunov equation
    # \[ AXE^T + EXAT + Q = 0 \]
    # where \( Q \) is a symmetric matrix and \( A, Q \) and \( E \) are square matrices of the same dimension.

### 3.7.5 control ctrb

control ctrb(A, B)

Controllability matrix

**Parameters**

\( B(A,\cdot) \) – Dynamics and input matrix of the system

**Returns**

\( C \) – Controllability matrix

**Return type**

matrix

**Examples**

```python
>>> C = ctrb(A, B)
```

### 3.7.6 control obsv

control obsv(A, C)

Observability matrix

**Parameters**

\( C(A,\cdot) \) – Dynamics and output matrix of the system

**Returns**

\( O \) – Observability matrix

**Return type**

matrix

**Examples**

```python
>>> O = obsv(A, C)
```
### 3.7.7 control.gram

**control.gram**(sys, type)

Gramian (controllability or observability)

**Parameters**

- **sys** (*StateSpace*) – State-space system to compute Gramian for
- **type** (*String*) – Type of desired computation. *type* is either 'c' (controllability) or 'o' (observability). To compute the Cholesky factors of gramians use 'cf' (controllability) or 'of' (observability)

**Returns**

- **gram** – Gramian of system

**Return type**

array

**Raises**

- **ValueError** – * if system is not instance of StateSpace class * if type is not 'c', 'o', 'cf' or 'of' * if system is unstable (sys.A has eigenvalues not in left half plane)
- **ImportError** – if slycot routine sb03md cannot be found if slycot routine sb03od cannot be found

**Examples**

```python
>>> Wc = gram(sys, 'c')
>>> Wo = gram(sys, 'o')
>>> Rc = gram(sys, 'cf'), where Wc=Rc'*Rc
>>> Ro = gram(sys, 'of'), where Wo=Ro'*Ro
```

### 3.8 Control system synthesis

#### 3.8.1 control.acker

**control.acker**(A, B, poles)

Pole placement using Ackermann method

**Parameters**

- **A** – State and input matrix of the system
- **B** – (A, ) – State and input matrix of the system
- **poles** (*1-d list*) – Desired eigenvalue locations

**Returns**

- **K** – Gains such that A - B K has given eigenvalues

**Return type**

matrix
3.8.2 control.h2syn

control.h2syn(P, nmeas, ncon)
H_2 control synthesis for plant P.

Parameters

- P (partitioned lti plant (State-space sys)) –
- nmeas (number of measurements (input to controller)) –
- ncon (number of control inputs (output from controller)) –

Returns K

Return type  controller to stabilize P (State-space sys)

Raises ImportError – if slycot routine sb10hd is not loaded

See also:
StateSpace()

Examples

```python
>>> K = h2syn(P, nmeas, ncon)
```

3.8.3 control.hinfsyn

control.hinfsyn(P, nmeas, ncon)
H_{inf} control synthesis for plant P.

Parameters

- P (partitioned lti plant) –
- nmeas (number of measurements (input to controller)) –
- ncon (number of control inputs (output from controller)) –

Returns

- K (controller to stabilize P (State-space sys))
- CL (closed loop system (State-space sys))
- gam (infinity norm of closed loop system)
- rcond (4-vector, reciprocal condition estimates of: 1: control transformation matrix 2: measurement transformation matrix 3: X-Ricatti equation 4: Y-Ricatti equation)
- TODO (document significance of rcond)

Raises ImportError – if slycot routine sb10ad is not loaded

See also:
StateSpace()
Examples

```python
>>> K, CL, gam, rcond = hinfsyn(P, nmeas, ncon)
```

### 3.8.4 control.lqr

**control.lqr** \((A, B, Q, R, [N])\)

Linear quadratic regulator design

The lqr() function computes the optimal state feedback controller that minimizes the quadratic cost

\[
J = \int_0^\infty (x'Qx + u'Ru + 2x'Nu)dt
\]

The function can be called with either 3, 4, or 5 arguments:

- lqr(sys, Q, R)
- lqr(sys, Q, R, N)
- lqr(A, B, Q, R)
- lqr(A, B, Q, R, N)

where `sys` is an LTI object, and `A`, `B`, `Q`, `R`, and `N` are 2d arrays or matrices of appropriate dimension.

**Parameters**

- **B** \((A,)\) – Dynamics and input matrices
- **sys** \((LTI \ (StateSpace \ or \ TransferFunction))\) – Linear I/O system
- **R** \((Q,)\) – State and input weight matrices
- **N** \((2-d \ array, \ optional)\) – Cross weight matrix

**Returns**

- **K** \((2-d \ array)\) – State feedback gains
- **S** \((2-d \ array)\) – Solution to Riccati equation
- **E** \((1-d \ array)\) – Eigenvalues of the closed loop system

Examples

```python
>>> K, S, E = lqr(sys, Q, R, [N])
>>> K, S, E = lqr(A, B, Q, R, [N])
```

### 3.8.5 control.mixsyn

**control.mixsyn** \((g, \ w1=\texttt{None}, \ w2=\texttt{None}, \ w3=\texttt{None})\)

Mixed-sensitivity H-infinity synthesis.

mixsyn(g, w1, w2, w3) -> k, cl, info

**Parameters**

- **g** \((LTI; \ the \ plant \ for \ which \ controller \ must \ be \ synthesized)\) –
• \( w_1 \) (weighting on \( s = (1+g*k)^{-1}; \) None, or scalar or \( k_1 \)-by-\( n_y \) LTI)-

• \( w_2 \) (weighting on \( k*s; \) None, or scalar or \( k_2 \)-by-\( n_u \) LTI)-

• \( w_3 \) (weighting on \( t = g*k*(1+g*k)^{-1}; \) None, or scalar or \( k_3 \)-by-\( n_y \) LTI)-

• least one of \( w_1 \), \( w_2 \), and \( w_3 \) must not be None. (At)-

Returns

• \( k \) (synthesized controller; StateSpace object)

• \( cl \) (closed system mapping evaluation inputs to evaluation outputs; if)

• \( p \) is the augmented plant, with – \( [z] = [p_{11} \ p_{12}] \ [w], \ [y] [p_{21} \ g] \ [u] \)

• then \( cl \) is the system from \( w->z \) with \( u=-k*y \). StateSpace object.

• \( info \) (tuple with entries, in order) –
  – gamma: scalar; H-infinity norm of \( cl \)
  – rcond: array; estimates of reciprocal condition numbers computed during synthesis. See hinf for details

• If a weighting \( w \) is scalar, it will be replaced by \( I^w \), where \( I \) is

• \( n_y \)-by-\( n_y \) for \( w_1 \) and \( w_3 \), and \( n_u \)-by-\( n_u \) for \( w_2 \).

See also:

\( \text{hinf}(), \text{augw}() \)

3.8.6 control.place

control.place(\( A, B, p \))
Place closed loop eigenvalues \( K = \text{place}(A, B, p) \)

Parameters

• \( A \) (2-d array) – Dynamics matrix

• \( B \) (2-d array) – Input matrix

• \( p \) (1-d list) – Desired eigenvalue locations

Returns

• \( K \) (2-d array) – Gain such that \( A - B \ K \) has eigenvalues given in \( p \)

• Algorithm

  • This is a wrapper function for scipy.signal.place_poles, which
  • implements the Tits and Yang algorithm [1]. It will handle SISO,
  • MISO, and MIMO systems. If you want more control over the algorithm,
  • use scipy.signal.place_poles directly.

  pole assignment by state feedback, IEEE Transactions on Automatic

• Limitations

• The algorithm will not place poles at the same location more than rank(B) times.

Examples

```python
>>> A = [[-1, -1], [0, 1]]
>>> B = [[0], [1]]
>>> K = place(A, B, [-2, -5])
```

See also:

place_varga(), acker()

3.9 Model simplification tools

<table>
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<tr>
<th>Function</th>
<th>Description</th>
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<tbody>
<tr>
<td><code>minreal(sys[, tol, verbose])</code></td>
<td>Eliminates uncontrollable or unobservable states in state-space models or cancelling pole-zero pairs in transfer functions.</td>
</tr>
<tr>
<td><code>balred(sys, orders[, method, alpha])</code></td>
<td>Balanced reduced order model of sys of a given order.</td>
</tr>
<tr>
<td><code>hsvd(sys)</code></td>
<td>Calculate the Hankel singular values.</td>
</tr>
<tr>
<td><code>modred(sys, ELIM[, method])</code></td>
<td>Model reduction of sys by eliminating the states in ELIM using a given method.</td>
</tr>
<tr>
<td><code>era(YY, m, n, nin, nout, r)</code></td>
<td>Calculate an ERA model of order r based on the impulse-response data YY.</td>
</tr>
<tr>
<td><code>markov(Y, U, M)</code></td>
<td>Calculate the first M Markov parameters [D CB CAB …] from input U, output Y.</td>
</tr>
</tbody>
</table>

3.9.1 control.minreal

`control.minreal (sys, tol=None, verbose=True)`

Eliminates uncontrollable or unobservable states in state-space models or cancelling pole-zero pairs in transfer functions. The output sysr has minimal order and the same response characteristics as the original model sys.

Parameters

• `sys` (StateSpace or TransferFunction) – Original system

• `tol` (real) – Tolerance

• `verbose` (bool) – Print results if True

Returns `rsys` – Cleaned model

Return type StateSpace or TransferFunction
3.9.2 control.balred

control.balred(sys, orders, method='truncate', alpha=None)
Balanced reduced order model of sys of a given order. States are eliminated based on Hankel singular value. If
sys has unstable modes, they are removed, the balanced realization is done on the stable part, then reinserted in
accordance with the reference below.

Electronics Letters, 27, 984-986.

Parameters

• sys (StateSpace) – Original system to reduce
• orders (integer or array of integer) – Desired order of reduced order model
  (if a vector, returns a vector of systems)
• method (string) – Method of removing states, either 'truncate' or 'matchdc'.
• alpha (float) – Redefines the stability boundary for eigenvalues of the system matrix
  A. By default for continuous-time systems, alpha <= 0 defines the stability boundary for
  the real part of A's eigenvalues and for discrete-time systems, 0 <= alpha <= 1 defines the
  stability boundary for the modulus of A's eigenvalues. See SLICOT routines AB09MD and
  AB09ND for more information.

Returns rsys – A reduced order model or a list of reduced order models if orders is a list

Return type StateSpace

 Raises

• ValueError – if method is not 'truncate' or 'matchdc'
• ImportError – if slycot routine ab09ad, ab09md, or ab09nd is not found
• ValueError – if there are more unstable modes than any value in orders

Examples

>>> rsys = balred(sys, orders, method='truncate')

3.9.3 control.hsvd

control.hsvd(sys)
Calculate the Hankel singular values.

Parameters sys (StateSpace) – A state space system

Returns H – A list of Hankel singular values

Return type Matrix

See also:
gram()
Notes

The Hankel singular values are the singular values of the Hankel operator. In practice, we compute the square root of the eigenvalues of the matrix formed by taking the product of the observability and controllability gramians. There are other (more efficient) methods based on solving the Lyapunov equation in a particular way (more details soon).

Examples

```python
>>> H = hsvd(sys)
```

3.9.4 control.modred

control.modred(sys, ELIM, method='matchdc')

Model reduction of `sys` by eliminating the states in `ELIM` using a given method.

Parameters

- `sys` (StateSpace) – Original system to reduce
- `ELIM` (array) – Vector of states to eliminate
- `method` (string) – Method of removing states in `ELIM`: either 'truncate' or 'matchdc'.

Returns `rsys` – A reduced order model

Return type `StateSpace`

Raises `ValueError` – Raised under the following conditions:

- if `method` is not either 'matchdc' or 'truncate'
- if eigenvalues of `sys.A` are not all in left half plane (`sys` must be stable)

Examples

```python
>>> rsys = modred(sys, ELIM, method='truncate')
```

3.9.5 control.era

control.era(YY, m, n, nin, nout, r)

Calculate an ERA model of order `r` based on the impulse-response data `YY`.

Note: This function is not implemented yet.

Parameters

- `YY` (array) – `nout` x `nin` dimensional impulse-response data
- `m` (integer) – Number of rows in Hankel matrix
- `n` (integer) – Number of columns in Hankel matrix
• **nin** (*integer*) – Number of input variables
• **nout** (*integer*) – Number of output variables
• **r** (*integer*) – Order of model

**Returns**  
`sys` – A reduced order model `sys=ss(Ar,Br,Cr,Dr)`

**Return type**  
`StateSpace`

**Examples**

```python
>>> rsys = era(YY, m, n, nin, nout, r)
```

### 3.9.6 control.markov

```python
def markov(Y, U, M):
    Calculate the first `M` Markov parameters `[D CB CAB ... ]` from input `U`, output `Y`.
```

**Parameters**

- **Y** (*array_like*) – Output data
- **U** (*array_like*) – Input data
- **M** (*integer*) – Number of Markov parameters to output

**Returns**  
`H` – First `M` Markov parameters

**Return type**  
`matrix`

**Notes**

Currently only works for SISO

**Examples**

```python
>>> H = markov(Y, U, M)
```

### 3.10 Nonlinear system support

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<td><code>find_eqpt</code></td>
<td>Find the equilibrium point for an input/output system.</td>
</tr>
<tr>
<td><code>linearize</code></td>
<td>Linearize an input/output system at a given state and input.</td>
</tr>
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</table>

#### 3.10.1 control.find_eqpt

```python
def find_eqpt(sys, x0=[], u0=[], y0=..., t=0, params=[], iu=None, iy=None, ix=None, idy=None, dx0=..., return_y=True, return_result=True, **kw)
    Find the equilibrium point for an input/output system.
```

Returns the value of an equilibrium point given the initial state and either input value or desired output value for
the equilibrium point.

**Parameters**

- **x0** *(list of initial state values)* – Initial guess for the value of the state near the equilibrium point.

- **u0** *(list of input values, optional)* – If `y0` is not specified, sets the equilibrium value of the input. If `y0` is given, provides an initial guess for the value of the input. Can be omitted if the system does not have any inputs.

- **y0** *(list of output values, optional)* – If specified, sets the desired values of the outputs at the equilibrium point.

- **t** *(float, optional)* – Evaluation time, for time-varying systems

- **params** *(dict, optional)* – Parameter values for the system. Passed to the evaluation functions for the system as default values, overriding internal defaults.

- **iu** *(list of input indices, optional)* – If specified, only the inputs with the given indices will be fixed at the specified values in solving for an equilibrium point. All other inputs will be varied. Input indices can be listed in any order.

- **iy** *(list of output indices, optional)* – If specified, only the outputs with the given indices will be fixed at the specified values in solving for an equilibrium point. All other outputs will be varied. Output indices can be listed in any order.

- **ix** *(list of state indices, optional)* – If specified, states with the given indices will be fixed at the specified values in solving for an equilibrium point. All other states will be varied. State indices can be listed in any order.

- **dx0** *(list of update values, optional)* – If specified, the value of update map must match the listed value instead of the default value of 0.

- **idx** *(list of state indices, optional)* – If specified, state updates with the given indices will have their update maps fixed at the values given in `dx0`. All other update values will be ignored in solving for an equilibrium point. State indices can be listed in any order. By default, all updates will be fixed at `dx0` in searching for an equilibrium point.

- **return_y** *(bool, optional)* – If True, return the value of output at the equilibrium point.

- **return_result** *(bool, optional)* – If True, return the `result` option from the `scipy root` function used to compute the equilibrium point.

**Returns**

- **xeq** *(array of states)* – Value of the states at the equilibrium point, or `None` if no equilibrium point was found and `return_result` was False.

- **ueq** *(array of input values)* – Value of the inputs at the equilibrium point, or `None` if no equilibrium point was found and `return_result` was False.

- **yeq** *(array of output values, optional)* – If `return_y` is True, returns the value of the outputs at the equilibrium point, or `None` if no equilibrium point was found and `return_result` was False.

- **result** *(scipy root() result object, optional)* – If `return_result` is True, returns the `result` from the `scipy root` function.
3.10.2 control.linearize

control.linearize(sys, xeq=[], ueq=[], t=0, params={}, **kw)
Linearize an input/output system at a given state and input.

This function computes the linearization of an input/output system at a given state and input value and returns a control.StateSpace object. The evaluation point need not be an equilibrium point.

Parameters

- **sys** (InputOutputSystem) – The system to be linearized
- **xeq** (array) – The state at which the linearization will be evaluated (does not need to be an equilibrium state).
- **ueq** (array) – The input at which the linearization will be evaluated (does not need to correspond to an equilibrium state).
- **t** (float, optional) – The time at which the linearization will be computed (for time-varying systems).
- **params** (dict, optional) – Parameter values for the systems. Passed to the evaluation functions for the system as default values, overriding internal defaults.

Returns **ss_sys** – The linearization of the system, as a LinearIOSystem object (which is also a StateSpace object).

Return type LinearIOSystem

3.11 Utility functions and conversions

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</table>
3.11.1 control.augw

control.augw(g, w1=None, w2=None, w3=None)
Augment plant for mixed sensitivity problem.

Parameters

• g (LTI object, ny-by-nu)–
• w1 (weighting on S; None, scalar, or k1-by-ny LTI object)–
• w2 (weighting on KS; None, scalar, or k2-by-nu LTI object)–
• w3 (weighting on T; None, scalar, or k3-by-ny LTI object)–
• p (augmented plant; StateSpace object)–
• a weighting is None, no augmentation is done for it. At least (If)–
• weighting must not be None. (one)–
• a weighting w is scalar, it will be replaced by I*w, where I is (If)–
• for w1 and w3, and nu-by-nu for w2: (ny-by-ny)–

Returns p

Return type plant augmented with weightings, suitable for submission to hinfsyn or h2syn.

Raises ValueError – if all weightings are None

See also:

h2syn(), hinfsyn(), mixsyn()

3.11.2 control.canonical_form

control.canonical_form(xsys, form='reachable')
Convert a system into canonical form

Parameters

• xsys (StateSpace object) – System to be transformed, with state ‘x’
• form (String) –

Canonical form for transformation. Chosen from:

– ’reachable’ - reachable canonical form
– ’observable’ - observable canonical form
– ’modal’ - modal canonical form

Returns

• zsys (StateSpace object) – System in desired canonical form, with state ‘z’
• T (matrix) – Coordinate transformation matrix, z = T * x
3.11.3 control.damp

control.damp(sys, doprint=True)

Compute natural frequency, damping ratio, and poles of a system
The function takes 1 or 2 parameters

Parameters

• sys (LTI (StateSpace or TransferFunction)) – A linear system object
• doprint – if true, print table with values

Returns

• wn (array) – Natural frequencies of the poles
• damping (array) – Damping values
• poles (array) – Pole locations

Algorithm

• If the system is continuous, wn = abs(poles) Z = -real(poles)/poles.
• If the system is discrete, the discrete poles are mapped to their equivalent location in the s-plane via s = log10(poles)/dt
• and wn = abs(s) Z = -real(s)/wn.

See also:

pole()

3.11.4 control.db2mag

control.db2mag(db)

Convert a gain in decibels (dB) to a magnitude
If A is magnitude,

db = 20 * log10(A)

Parameters db (float or ndarray) – input value or array of values, given in decibels

Returns mag – corresponding magnitudes

Return type float or ndarray

3.11.5 control.isctime

control.isctime(sys, strict=False)

Check to see if a system is a continuous-time system

Parameters

• sys (LTI system) – System to be checked
• strict (bool (default = False)) – If strict is True, make sure that timebase is not None
3.11.6 control.isdtime

control.isdtime(sys, strict=False)
Check to see if a system is a discrete time system

Parameters

• sys (LTI system) – System to be checked
• strict (bool (default = False)) – If strict is True, make sure that timebase is not None

3.11.7 control.issiso

control.issiso(sys, strict=False)
Check to see if a system is single input, single output

Parameters

• sys (LTI system) – System to be checked
• strict (bool (default = False)) – If strict is True, do not treat scalars as SISO

3.11.8 control.issys

control.issys(obj)
Return True if an object is a system, otherwise False

3.11.9 control.mag2db

control.mag2db(mag)
Convert a magnitude to decibels (dB)

If A is magnitude,
   \[ \text{db} = 20 \times \log_{10}(A) \]

Parameters mag (float or ndarray) – input magnitude or array of magnitudes
Returns db – corresponding values in decibels
Return type float or ndarray

3.11.10 control.observable_form

control.observable_form(xsys)
Convert a system into observable canonical form

Parameters xsys (StateSpace object) – System to be transformed, with state x

Returns

• zsys (StateSpace object) – System in observable canonical form, with state z
• T (matrix) – Coordinate transformation: z = T * x
3.11.11 control.pade

control.pade(T, n=1, numdeg=None)
Create a linear system that approximates a delay.

Return the numerator and denominator coefficients of the Pade approximation.

Parameters

- T (number) – time delay
- n (positive integer) – degree of denominator of approximation
- numdeg (integer, or None (the default)) – If None, numerator degree equals denominator degree If >= 0, specifies degree of numerator If < 0, numerator degree is n+numdeg

Returns num, den – Polynomial coefficients of the delay model, in descending powers of s.

Return type array

Notes
Based on:
1. Algorithm 11.3.1 in Golub and van Loan, “Matrix Computation” 3rd. Ed. pp. 572-574

3.11.12 controlreachable_form

controlreachable_form(xsys)
Convert a system into reachable canonical form

Parameters xsys (StateSpace object) – System to be transformed, with state x

Returns

- zsys (StateSpace object) – System in reachable canonical form, with state z
- T (matrix) – Coordinate transformation: \( z = T * x \)

3.11.13 control.sample_system

control.sample_system(sysc, Ts, method='zoh', alpha=None)
Convert a continuous time system to discrete time

Creates a discrete time system from a continuous time system by sampling. Multiple methods of conversion are supported.

Parameters

- sysc (linsys) – Continuous time system to be converted
- Ts (real) – Sampling period
- method (string) – Method to use for conversion: ‘matched’, ‘tustin’, ‘zoh’ (default)

Returns sysd – Discrete time system, with sampling rate Ts

Return type linsys
Notes

See `TransferFunction.sample` and `StateSpace.sample` for further details.

Examples

```python
>>> sysc = TransferFunction([1], [1, 2, 1])
>>> sysd = sample_system(sysc, 1, method='matched')
```

3.11.14 control.ss2tf

ccontrol.ss2tf(sys)

Transform a state space system to a transfer function.

The function accepts either 1 or 4 parameters:

- **ss2tf(sys)** Convert a linear system into space system form. Always creates a new system, even if sys is already a StateSpace object.
- **ss2tf(A, B, C, D)** Create a state space system from the matrices of its state and output equations.

For details see: `ss()`

Parameters

- **sys** (StateSpace) – A linear system
- **A** (array_like or string) – System matrix
- **B** (array_like or string) – Control matrix
- **C** (array_like or string) – Output matrix
- **D** (array_like or string) – Feedthrough matrix

Returns **out** – New linear system in transfer function form

Return type **TransferFunction**

Raises

- **ValueError** – if matrix sizes are not self-consistent, or if an invalid number of arguments is passed in
- **TypeError** – if `sys` is not a StateSpace object

See also:

`tf(), ss(), tf2ss()`

Examples

```python
>>> A = [[1., -2], [3, -4]]
>>> B = [[5.], [7]]
>>> C = [[6., 8]]
>>> D = [[9.]]
>>> sys1 = ss2tf(A, B, C, D)
```
**3.11.15 control.ssdata**

`control.ssdata(sys)`

Return state space data objects for a system

**Parameters**
- `sys` (*LTI (StateSpace, or TransferFunction)*) – LTI system whose data will be returned

**Returns**
- `(A, B, C, D)` – State space data for the system

**Return type** list of matrices

**3.11.16 control.tf2ss**

`control.tf2ss(sys)`

Transform a transfer function to a state space system.

The function accepts either 1 or 2 parameters:

- `tf2ss(sys)` Convert a linear system into transfer function form. Always creates a new system, even if `sys` is already a TransferFunction object.

- `tf2ss(num, den)` Create a transfer function system from its numerator and denominator polynomial coefficients.

  For details see: `tf()`

**Parameters**

- `sys` (*LTI (StateSpace or TransferFunction)*) – A linear system
- `num` (*array_like, or list of list of array_like*) – Polynomial coefficients of the numerator
- `den` (*array_like, or list of list of array_like*) – Polynomial coefficients of the denominator

**Returns**
- `out` – New linear system in state space form

**Return type** *StateSpace*

**Raises**

- `ValueError` – if `num` and `den` have invalid or unequal dimensions, or if an invalid number of arguments is passed in
- `TypeError` – if `num` or `den` are of incorrect type, or if `sys` is not a TransferFunction object

**See also:**

`ss(), tf(), ss2tf()`
Examples

```python
>>> num = [[[1., 2.], [3., 4.]], [[5., 6.], [7., 8.]]]
>>> den = [[[9., 8., 7.], [6., 5., 4.]], [[3., 2., 1.], [-1., -2., -3.]]]
>>> sys1 = tf2ss(num, den)

>>> sys_tf = tf(num, den)
>>> sys2 = tf2ss(sys_tf)
```

### 3.11.17 control.tfdata

**control.tfdata**(sys)

Return transfer function data objects for a system

- **Parameters**
  - **sys** (*LTI (StateSpace, or TransferFunction)*) – LTI system whose data will be returned

- **Returns**
  - **(num, den)** – Transfer function coefficients (SISO only)

- **Return type** numerical and denominator arrays

### 3.11.18 control.timebase

**control.timebase**(sys, strict=True)

Return the timebase for an LTI system

```python
dt = timebase(sys)
```

returns the timebase for a system `sys`. If the strict option is set to False, `dt = True` will be returned as 1.

### 3.11.19 control.timebaseEqual

**control.timebaseEqual**(sys1, sys2)

Check to see if two systems have the same timebase

```python
timebaseEqual(sys1, sys2)
```

returns True if the timebases for the two systems are compatible. By default, systems with timebase ‘None’ are compatible with either discrete or continuous timebase systems. If two systems have a discrete timebase (dt > 0) then their timebases must be equal.

### 3.11.20 control.unwrap

**control.unwrap**(angle, period=6.283185307179586)

Unwrap a phase angle to give a continuous curve

- **Parameters**
  - **angle** (*array_like*) – Array of angles to be unwrapped
  - **period** (*float, optional*) – Period (defaults to 2*pi)

- **Returns**
  - **angle_out** – Output array, with jumps of period/2 eliminated

- **Return type** array_like
Examples

```python
>>> import numpy as np
>>> theta = [5.74, 5.97, 6.19, 0.13, 0.35, 0.57]
>>> unwrap(theta, period=2 * np.pi)
[5.74, 5.97, 6.19, 6.413185307179586, 6.633185307179586, 6.8531853071795865]
```
The classes listed below are used to represent models of linear time-invariant (LTI) systems. They are usually created from factory functions such as \texttt{tf()} and \texttt{ss()}, so the user should normally not need to instantiate these directly.

- \texttt{TransferFunction(num, den[, dt])}  
  A class for representing transfer functions

- \texttt{StateSpace(A, B, C, D[, dt])}  
  A class for representing state-space models

- \texttt{FRD(d, w)}  
  A class for models defined by frequency response data (FRD)

- \texttt{InputOutputSystem([inputs, outputs, states, ...])}  
  A class for representing input/output systems.

### 4.1 \texttt{control.TransferFunction}

\begin{verbatim}
class control.TransferFunction(num, den[, dt])  
A class for representing transfer functions

The TransferFunction class is used to represent systems in transfer function form.

The main data members are ‘num’ and ‘den’, which are 2-D lists of arrays containing MIMO numerator and denominator coefficients. For example,

```python
>>> num[2][5] = numpy.array([1., 4., 8.])
```

means that the numerator of the transfer function from the 6th input to the 3rd output is set to \(s^2 + 4s + 8\).

Discrete-time transfer functions are implemented by using the ‘dt’ instance variable and setting it to something other than ‘None’. If ‘dt’ has a non-zero value, then it must match whenever two transfer functions are combined. If ‘dt’ is set to True, the system will be treated as a discrete time system with unspecified sampling time.

The TransferFunction class defines two constants \(s\) and \(z\) that represent the differentiation and delay operators in continuous and discrete time. These can be used to create variables that allow algebraic creation of transfer functions. For example,
```python
>>> s = TransferFunction.s
>>> G = (s + 1)/(s**2 + 2*s + 1)
```

```python
__init__(*)
TransferFunction(num, den[, dt])
```

Construct a transfer function.

The default constructor is `TransferFunction(num, den)`, where `num` and `den` are lists of lists of arrays containing polynomial coefficients. To create a discrete time transfer function, use `TransferFunction(num, den, dt)` where `dt` is the sampling time (or `True` for unspecified sampling time). To call the copy constructor, call `TransferFunction(sys)`, where `sys` is a `TransferFunction` object (continuous or discrete).

### Methods

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<td>dcgain()</td>
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### Attributes

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#### damping

Natural frequency, damping ratio of system poles

**Returns**

- **wn** *(array)* – Natural frequencies for each system pole
- **zeta** *(array)* – Damping ratio for each system pole
- **poles** *(array)* – Array of system poles

#### dcgain()

Return the zero-frequency (or DC) gain
For a continuous-time transfer function \( G(s) \), the DC gain is \( G(0) \). For a discrete-time transfer function \( G(z) \), the DC gain is \( G(1) \).

**Returns** gain – The zero-frequency gain

**Return type** ndarray

**evalfr** (*omega*)
Evaluate a transfer function at a single angular frequency.

self._evalfr(omega) returns the value of the transfer function matrix with input value \( s = i \times \omega \).

**feedback** (*other=1, sign=-1*)
Feedback interconnection between two LTI objects.

**freqresp** (*omega*)
Evaluate a transfer function at a list of angular frequencies.

mag, phase, omega = self.freqresp(omega)
reports the value of the magnitude, phase, and angular frequency of the transfer function matrix evaluated at \( s = i \times \omega \), where omega is a list of angular frequencies, and is a sorted version of the input omega.

**horner** (*s*)
Evaluate the system’s transfer function for a complex variable

Returns a matrix of values evaluated at complex variable \( s \).

**isctime** (*strict=False*)
Check to see if a system is a continuous-time system

Parameters
- **sys** (*LTI system*) – System to be checked
- **strict** (*bool, optional*) – If strict is True, make sure that timebase is not None. Default is False.

**isdttime** (*strict=False*)
Check to see if a system is a discrete-time system

Parameters **strict** (*bool, optional*) – If strict is True, make sure that timebase is not None. Default is False.

**issiso**()
Check to see if a system is single input, single output

**minreal** (*tol=None*)
Remove cancelling pole/zero pairs from a transfer function

**pole**()
Compute the poles of a transfer function.

**returnScipySignalLTI**()
Return a list of a list of scipy.signal.lti objects.

For instance,

```python
>>> out = tfobject.returnScipySignalLTI()
>>> out[3][5]
```

is a signal.scipy.lti object corresponding to the transfer function from the 6th input to the 4th output.

**sample** (*Ts, method='zoh', alpha=None*)
Convert a continuous-time system to discrete time
Creates a discrete-time system from a continuous-time system by sampling. Multiple methods of conversion are supported.

**Parameters**

- **Ts** (*float*) – Sampling period
- **method** (*{'gbt', 'bilinear', 'euler', 'backward_diff', 'zoh', 'matched'}*) Method to use for sampling:
  - gbt: generalized bilinear transformation
  - bilinear: Tustin’s approximation (“gbt” with alpha=0.5)
  - euler: Euler (or forward difference) method (“gbt” with alpha=0)
  - backward_diff: Backwards difference (“gbt” with alpha=1.0)
  - zoh: zero-order hold (default)
- **alpha** (*float within [0, 1]*) – The generalized bilinear transformation weighting parameter, which should only be specified with method=“gbt”, and is ignored otherwise.

**Returns**

- **sysd** – Discrete time system, with sampling rate Ts

**Return type** StateSpace system

**Notes**

1. Available only for SISO systems
2. Uses the command `cont2discrete` from `scipy.signal`

**Examples**

```python
>>> sys = TransferFunction(1, [1,1])
>>> sysd = sys.sample(0.5, method='bilinear')
```

**zero()**

Compute the zeros of a transfer function.

### 4.2 control.StateSpace

**class** `control.StateSpace(A, B, C[, D[, dt]])`

A class for representing state-space models

The StateSpace class is used to represent state-space realizations of linear time-invariant (LTI) systems:

\[
\frac{dx}{dt} = Ax + Bu \quad y = Cx + Du
\]

where \( u \) is the input, \( y \) is the output, and \( x \) is the state.

The main data members are the A, B, C, and D matrices. The class also keeps track of the number of states (i.e., the size of A).

Discrete-time state space system are implemented by using the ‘dt’ instance variable and setting it to the sampling period. If ‘dt’ is not None, then it must match whenever two state space systems are combined. Setting dt
= 0 specifies a continuous system, while leaving \( dt = \text{None} \) means the system timebase is not specified. If ‘dt’ is set to True, the system will be treated as a discrete time system with unspecified sampling time.

```python
__init__(\*args, **kw)
StateSpace(A, B, C, D[, dt])
```

Construct a state space object.

The default constructor is `StateSpace(A, B, C, D)`, where \( A, B, C, D \) are matrices or equivalent objects. To create a discrete time system, use `StateSpace(A, B, C, D, dt)` where ‘dt’ is the sampling time (or True for unspecified sampling time). To call the copy constructor, call `StateSpace(sys)`, where `sys` is a `StateSpace` object.

### Methods

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<td>StateSpace(A, B, C, D[, dt])</td>
</tr>
<tr>
<td>append</td>
<td>Append a second model to the present model.</td>
</tr>
<tr>
<td>damp</td>
<td>Natural frequency, damping ratio of system poles</td>
</tr>
<tr>
<td>dcmgain</td>
<td>Return the zero-frequency gain</td>
</tr>
<tr>
<td>evalfr</td>
<td>Evaluate a SS system’s transfer function at a single frequency.</td>
</tr>
<tr>
<td>feedback</td>
<td>Feedback interconnection between two LTI systems.</td>
</tr>
<tr>
<td>freqresp</td>
<td>Evaluate the system’s transfer func.</td>
</tr>
<tr>
<td>horner</td>
<td>Evaluate the systems’s transfer function for a complex variable</td>
</tr>
<tr>
<td>isctime</td>
<td>Check to see if a system is a continuous-time system</td>
</tr>
<tr>
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<tr>
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<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>returnScipySignalLTI</td>
<td>Return a list of a list of scipy.signal.lti objects.</td>
</tr>
<tr>
<td>sample</td>
<td>Convert a continuous time system to discrete time</td>
</tr>
<tr>
<td>zero</td>
<td>Compute the zeros of a state space system.</td>
</tr>
</tbody>
</table>

### append

Append a second model to the present model. The second model is converted to state-space if necessary, inputs and outputs are appended and their order is preserved.

### damp

Natural frequency, damping ratio of system poles

**Returns**

- **wn** *(array)* – Natural frequencies for each system pole
- **zeta** *(array)* – Damping ratio for each system pole
- **poles** *(array)* – Array of system poles

### dcmgain

Return the zero-frequency gain

The zero-frequency gain of a continuous-time state-space system is given by:

and of a discrete-time state-space system by:
Returns gain – An array of shape (outputs,inputs); the array will either be the zero-frequency (or DC) gain, or, if the frequency response is singular, the array will be filled with np.nan.

Return type ndarray

evalfr (omega)
Evaluate a SS system’s transfer function at a single frequency.
self._evalfr(omega) returns the value of the transfer function matrix with input value s = i * omega.

feedback (other=1, sign=-1)
Feedback interconnection between two LTI systems.

freqresp (omega)
Evaluate the system’s transfer func. at a list of freqs, omega.

mag, phase, omega = self.freqresp(omega)

Reports the frequency response of the system,

\[ G(j\omega) = \text{mag} \cdot e^{j\text{phase}} \]

for continuous time. For discrete time systems, the response is evaluated around the unit circle such that

\[ G(e^{j\omega dt}) = \text{mag} \cdot e^{j\text{phase}}. \]

Parameters omega (array) – A list of frequencies in radians/sec at which the system should be evaluated. The list can be either a python list or a numpy array and will be sorted before evaluation.

Returns

• mag (float) – The magnitude (absolute value, not dB or log10) of the system frequency response.

• phase (float) – The wrapped phase in radians of the system frequency response.

• omega (array) – The list of sorted frequencies at which the response was evaluated.

horner (s)
Evaluate the systems’s transfer function for a complex variable

Returns a matrix of values evaluated at complex variable s.

isctime (strict=False)
Check to see if a system is a continuous-time system

Parameters

• sys (LTI system) – System to be checked

• strict (bool, optional) – If strict is True, make sure that timebase is not None. Default is False.

isdtime (strict=False)
Check to see if a system is a discrete-time system

Parameters strict (bool, optional) – If strict is True, make sure that timebase is not None. Default is False.

issiso ()
Check to see if a system is single input, single output
**lft** *(other, nu=-1, ny=-1)*

Return the Linear Fractional Transformation.

A definition of the LFT operator can be found in Appendix A.7, page 512 in the 2nd Edition, Multivariable Feedback Control by Sigurd Skogestad.

An alternative definition can be found here: https://www.mathworks.com/help/control/ref/lft.html

**Parameters**

- **other** *(LTI)* – The lower LTI system
- **ny** *(int, optional)* – Dimension of (plant) measurement output.
- **nu** *(int, optional)* – Dimension of (plant) control input.

**minreal** *(tol=0.0)*

Calculate a minimal realization, removes unobservable and uncontrollable states

**pole** *

Compute the poles of a state space system.

**returnScipySignalLTI** *

Return a list of a list of scipy.signal.lti objects.

For instance,

```python
>>> out = ssobject.returnScipySignalLTI()
>>> out[3][5]
```

is a signal.scipy.lti object corresponding to the transfer function from the 6th input to the 4th output.

**sample** *(Ts, method='zoh', alpha=None)*

Convert a continuous time system to discrete time

Creates a discrete-time system from a continuous-time system by sampling. Multiple methods of conversion are supported.

**Parameters**

- **Ts** *(float)* – Sampling period
- **method** *("gbt", "bilinear", "euler", "backward_diff", "zoh")* – Which method to use:
  - **gbt**: generalized bilinear transformation
  - **bilinear**: Tustin’s approximation ("gbt" with alpha=0.5)
  - **euler**: Euler (or forward differencing) method ("gbt" with alpha=0)
  - **backward_diff**: Backwards differencing ("gbt" with alpha=1.0)
  - **zoh**: zero-order hold (default)
- **alpha** *(float within [0, 1])* – The generalized bilinear transformation weighting parameter, which should only be specified with method="gbt", and is ignored otherwise

**Returns** sysd – Discrete time system, with sampling rate Ts

**Return type** *StateSpace*
Notes

Uses the command ‘cont2discrete’ from scipy.signal

Examples

```python
>>> sys = StateSpace(0, 1, 1, 0)
>>> sysd = sys.sample(0.5, method='bilinear')
```

`zero()`

Compute the zeros of a state space system.

4.3 control.FRDPython Control Library Documentation, Release dev

class control.FRDPython Control Library Documentation, Release dev

class control.FRDr(d, w)

A class for models defined by frequency response data (FRD)

The FRD class is used to represent systems in frequency response data form.

The main data members are ‘omega’ and ‘fresp’, where omega is a 1D array with the frequency points of the
response, and fresp is a 3D array, with the first dimension corresponding to the output index of the FRD, the
second dimension corresponding to the input index, and the 3rd dimension corresponding to the frequency points
in omega. For example,

```python
>>> frdata[2,5,:] = numpy.array([1., 0.8-0.2j, 0.2-0.8j])
```

means that the frequency response from the 6th input to the 3rd output at the frequencies defined in omega is set
to the array above, i.e. the rows represent the outputs and the columns represent the inputs.

```python
__init__(*args, **kwargs) FrD(d, w)

Construct an FRD object

The default constructor is FRD(d, w), where w is an iterable of frequency points, and d is the matching
frequency data.

If d is a single list, 1d array, or tuple, a SISO system description is assumed. d can also be

To call the copy constructor, call FRD(sys), where sys is a FRD object.

To construct frequency response data for an existing LTI object, other than an FRD, call FRD(sys, omega)

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>init</strong>(*args, **kwargs)</td>
<td>FRD(d, w)</td>
</tr>
<tr>
<td>damp()</td>
<td>Natural frequency, damping ratio of system poles</td>
</tr>
<tr>
<td>dcgain()</td>
<td>Return the zero-frequency gain</td>
</tr>
</tbody>
</table>
| eval(omega)        | Evaluate a transfer function at a single angular fre-
|                    | quency.                                              |
| evalfr(omega)      | Evaluate a transfer function at a single angular fre-
|                    | quency.                                              |
| feedback([other, sign]) | Feedback interconnection between two FRD objects.    |

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Table 5 – continued from previous page

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>freqresp</code></td>
<td>Evaluate a transfer function at a list of angular frequencies.</td>
</tr>
<tr>
<td><code>isctime</code></td>
<td>Check to see if a system is a continuous-time system</td>
</tr>
<tr>
<td><code>isdtime</code></td>
<td>Check to see if a system is a discrete-time system</td>
</tr>
<tr>
<td><code>issiso</code></td>
<td>Check to see if a system is single input, single output</td>
</tr>
</tbody>
</table>

**Attributes**

- **epsw**

  - **damp()**
    - Natural frequency, damping ratio of system poles

  **Returns**
  - **wn (array)** – Natural frequencies for each system pole
  - **zeta (array)** – Damping ratio for each system pole
  - **poles (array)** – Array of system poles

- **dcgain()**
  - Return the zero-frequency gain

- **eval(omega)**
  - Evaluate a transfer function at a single angular frequency.
    - `self.evalfr(omega)` returns the value of the frequency response at frequency `omega`.
    - Note that a “normal” FRD only returns values for which there is an entry in the `omega` vector. An interpolating FRD can return intermediate values.

- **evalfr(omega)**
  - Evaluate a transfer function at a single angular frequency.
    - `self._evalfr(omega)` returns the value of the frequency response at frequency `omega`.
    - Note that a “normal” FRD only returns values for which there is an entry in the `omega` vector. An interpolating FRD can return intermediate values.

- **feedback(other=1, sign=-1)**
  - Feedback interconnection between two FRD objects.

- **freqresp(omega)**
  - Evaluate a transfer function at a list of angular frequencies.
    - `mag, phase, omega = self.freqresp(omega)` reports the value of the magnitude, phase, and angular frequency of the transfer function matrix evaluated at `s = i * omega`, where `omega` is a list of angular frequencies, and is a sorted version of the input `omega`.

- **isctime(strict=False)**
  - Check to see if a system is a continuous-time system

  **Parameters**
  - **sys (LTI system)** – System to be checked
  - **strict (bool, optional)** – If strict is True, make sure that timebase is not None. Default is False.
isdtimem (strict=False)
    Check to see if a system is a discrete-time system

    Parameters
    strict (bool, optional) – If strict is True, make sure that timebase is not None. Default is False.

issiso()
    Check to see if a system is single input, single output

4.4 control.InputOutputSystem

class control.InputOutputSystem(inputs=None, outputs=None, states=None, params={},
    dt=None, name=None)
    A class for representing input/output systems.

    The InputOutputSystem class allows (possibly nonlinear) input/output systems to be represented in Python. It is intended as a parent class for a set of subclasses that are used to implement specific structures and operations for different types of input/output dynamical systems.

    Parameters

    • inputs (int, list of str, or None) – Description of the system inputs. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form s[i] (where s is one of u, y, or x). If this parameter is not given or given as None, the relevant quantity will be determined when possible based on other information provided to functions using the system.

    • outputs (int, list of str, or None) – Description of the system outputs. Same format as inputs.

    • states (int, list of str, or None) – Description of the system states. Same format as inputs.

    • dt (None, True or float, optional) – System timebase. None (default) indicates continuous time, True indicates discrete time with undefined sampling time, positive number is discrete time with specified sampling time.

    • params (dict, optional) – Parameter values for the systems. Passed to the evaluation functions for the system as default values, overriding internal defaults.

    • name (string, optional) – System name (used for specifying signals)

ninputs, noutputs, nstates
    Number of input, output and state variables

    Type int

input_index, output_index, state_index
    Dictionary of signal names for the inputs, outputs and states and the index of the corresponding array

    Type dict

dt
    System timebase. None (default) indicates continuous time, True indicates discrete time with undefined sampling time, positive number is discrete time with specified sampling time.

    Type None, True or float
params
Parameter values for the systems. Passed to the evaluation functions for the system as default values, overriding internal defaults.

Type dict, optional

name
System name (used for specifying signals)

Type string, optional

Notes
The InputOutputSystem class (and its subclasses) makes use of two special methods for implementing much of the work of the class:

- `_rhs(t, x, u)`: compute the right hand side of the differential or difference equation for the system. This must be specified by the subclass for the system.
- `_out(t, x, u)`: compute the output for the current state of the system. The default is to return the entire system state.

__init__ (inputs=None, outputs=None, states=None, params={}, dt=None, name=None)
Create an input/output system.

The InputOutputSystem constructor is used to create an input/output object with the core information required for all input/output systems. Instances of this class are normally created by one of the input/output subclasses: LinearIOSystem, NonlinearIOSystem, InterconnectedSystem.

Parameters

- **inputs** (int, list of str, or None) – Description of the system inputs. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form s[i] (where s is one of u, y, or x). If this parameter is not given or given as None, the relevant quantity will be determined when possible based on other information provided to functions using the system.
- **outputs** (int, list of str, or None) – Description of the system outputs. Same format as inputs.
- **states** (int, list of str, or None) – Description of the system states. Same format as inputs.
- **dt** (None, True or float, optional) – System timebase. None (default) indicates continuous time. True indicates discrete time with undefined sampling time, positive number is discrete time with specified sampling time.
- **params** (dict, optional) – Parameter values for the systems. Passed to the evaluation functions for the system as default values, overriding internal defaults.
- **name** (string, optional) – System name (used for specifying signals)

Returns Input/output system object

Return type InputOutputSystem

Methods
Python Control Library Documentation, Release dev

__init__((inputs, outputs, states, params, ...))
Create an input/output system.

copy()
Make a copy of an input/output system.

feedback([other, sign, params])
Feedback interconnection between two input/output systems

find_input(name)
Find the index for an input given its name (None if not found)

find_output(name)
Find the index for an output given its name (None if not found)

find_state(name)
Find the index for a state given its name (None if not found)

linearize(x0, u0[, t, params, eps])
Linearize an input/output system at a given state and input.

set_inputs(inputs[, prefix])
Set the number/names of the system inputs.

set_outputs(outputs[, prefix])
Set the number/names of the system outputs.

set_states(states[, prefix])
Set the number/names of the system states.

copy()
Make a copy of an input/output system.

feedback(other=1, sign=-1, params={})
Feedback interconnection between two input/output systems

Parameters

• sys1 (InputOutputSystem) – The primary process.

• sys2 (InputOutputSystem) – The feedback process (often a feedback controller).

• sign (scalar, optional) – The sign of feedback. sign = -1 indicates negative feedback, and sign = 1 indicates positive feedback. sign is an optional argument; it assumes a value of -1 if not specified.

Returns out

Return type InputOutputSystem

Raises ValueError – if the inputs, outputs, or timebases of the systems are incompatible.

find_input(name)
Find the index for an input given its name (None if not found)

find_output(name)
Find the index for an output given its name (None if not found)

find_state(name)
Find the index for a state given its name (None if not found)

linearize(x0, u0[, t, params, eps])
Linearize an input/output system at a given state and input.

Return the linearization of an input/output system at a given state and input value as a StateSpace system.
See linearize() for complete documentation.

set_inputs(inputs, prefix='u')
Set the number/names of the system inputs.

Parameters

• inputs (int, list of str, or None) – Description of the system inputs. This can be given as an integer count or as a list of strings that name the individual signals. If
an integer count is specified, the names of the signal will be of the form $u[i]$ (where the
prefix $u$ can be changed using the optional prefix parameter).

- **prefix** *(string, optional)* – If inputs is an integer, create the names of the states
using the given prefix (default = ‘u’). The names of the input will be of the form $prefix[i]$.

```
set_outputs (outputs, prefix='y')
```

Set the number/names of the system outputs.

**Parameters**

- **outputs (int, list of str, or None)* – Description of the system outputs. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form $u[i]$ (where the prefix $u$ can be changed using the optional prefix parameter).

- **prefix (string, optional)* – If outputs is an integer, create the names of the states
using the given prefix (default = ‘u’). The names of the input will be of the form $prefix[i]$.

```
set_states (states, prefix='x')
```

Set the number/names of the system states.

**Parameters**

- **states (int, list of str, or None)* – Description of the system states. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form $u[i]$ (where the prefix $u$ can be changed using the optional prefix parameter).

- **prefix (string, optional)* – If states is an integer, create the names of the states
using the given prefix (default = ‘u’). The names of the input will be of the form $prefix[i]$.

## 4.5 Input/Output system subclasses

Input/output systems are accessed primarily via a set of subclasses that allow for linear, nonlinear, and interconnected elements:

<table>
<thead>
<tr>
<th>Class</th>
<th>Description</th>
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<tbody>
<tr>
<td>LinearIOSystem</td>
<td>Input/output representation of a linear (state space) system.</td>
</tr>
<tr>
<td>NonlinearIOSystem</td>
<td>Nonlinear I/O system.</td>
</tr>
<tr>
<td>InterconnectedSystem</td>
<td>Interconnection of a set of input/output systems.</td>
</tr>
</tbody>
</table>

### 4.5.1 control.LinearIOSystem

```
class control.LinearIOSystem (linsys, inputs=None, outputs=None, states=None, name=None)
```

Input/output representation of a linear (state space) system.

This class is used to implement a system that is a linear state space system (defined by the StateSpace system object).

```
__init__ (linsys, inputs=None, outputs=None, states=None, name=None)
```

Create an I/O system from a state space linear system.

Converts a StateSpace system into an InputOutputSystem with the same inputs, outputs, and states. The new system can be a continuous or discrete time system.
Parameters

- **linsys** *(StateSpace)* – LTI StateSpace system to be converted
- **inputs** *(int, list of str or None, optional)* – Description of the system inputs. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form $s[i]$ (where $s$ is one of $u$, $y$, or $x$). If this parameter is not given or given as `None`, the relevant quantity will be determined when possible based on other information provided to functions using the system.
- **outputs** *(int, list of str or None, optional)* – Description of the system outputs. Same format as `inputs`.
- **states** *(int, list of str, or None, optional)* – Description of the system states. Same format as `inputs`.
- **dt** *(None, True or float, optional)* – System timebase. None (default) indicates continuous time, True indicates discrete time with undefined sampling time, positive number is discrete time with specified sampling time.
- **params** *(dict, optional)* – Parameter values for the systems. Passed to the evaluation functions for the system as default values, overriding internal defaults.
- **name** *(string, optional)* – System name (used for specifying signals)

**Returns** `iosys` – Linear system represented as an input/output system

**Return type** `LinearIOSystem`

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>__init__</code>(linsys[, inputs, outputs, states, name])</td>
<td>Create an I/O system from a state space linear system.</td>
</tr>
<tr>
<td><code>append</code>(other)</td>
<td>Append a second model to the present model.</td>
</tr>
<tr>
<td><code>copy</code>()</td>
<td>Make a copy of an input/output system.</td>
</tr>
<tr>
<td><code>damp</code>()</td>
<td>Natural frequency, damping ratio of system poles</td>
</tr>
<tr>
<td><code>dcgain</code>()</td>
<td>Return the zero-frequency gain</td>
</tr>
<tr>
<td><code>evalfr</code>(omega)</td>
<td>Evaluate a SS system’s transfer function at a single frequency.</td>
</tr>
<tr>
<td><code>feedback</code>([other, sign, params])</td>
<td>Feedback interconnection between two input/output systems</td>
</tr>
<tr>
<td><code>find_input</code>(name)</td>
<td>Find the index for an input given its name (None if not found)</td>
</tr>
<tr>
<td><code>find_output</code>(name)</td>
<td>Find the index for an output given its name (None if not found)</td>
</tr>
<tr>
<td><code>find_state</code>(name)</td>
<td>Find the index for a state given its name (None if not found)</td>
</tr>
<tr>
<td><code>freqresp</code>(omega)</td>
<td>Evaluate the system’s transfer func.</td>
</tr>
<tr>
<td><code>horner</code>(s)</td>
<td>Evaluate the systems’s transfer function for a complex variable</td>
</tr>
<tr>
<td><code>isctime</code>([strict])</td>
<td>Check to see if a system is a continuous-time system</td>
</tr>
<tr>
<td><code>isdttime</code>([strict])</td>
<td>Check to see if a system is a discrete-time system</td>
</tr>
<tr>
<td><code>issiso</code>()</td>
<td>Check to see if a system is single input, single output</td>
</tr>
<tr>
<td><code>lft</code>(other[, nu, ny])</td>
<td>Return the Linear Fractional Transformation.</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>linearize(x0, u0[, t, params, eps])</code></td>
<td>Linearize an input/output system at a given state and input.</td>
</tr>
<tr>
<td><code>minreal([tol])</code></td>
<td>Calculate a minimal realization, removes unobservable and uncontrollable states</td>
</tr>
<tr>
<td><code>pole()</code></td>
<td>Compute the poles of a state space system.</td>
</tr>
<tr>
<td><code>returnScipySignalLTI()</code></td>
<td>Return a list of a list of scipy.signal.lti objects.</td>
</tr>
<tr>
<td><code>sample(Ts[, method, alpha])</code></td>
<td>Convert a continuous time system to discrete time</td>
</tr>
<tr>
<td><code>set_inputs(inputs[, prefix])</code></td>
<td>Set the number/names of the system inputs.</td>
</tr>
<tr>
<td><code>set_outputs(outputs[, prefix])</code></td>
<td>Set the number/names of the system outputs.</td>
</tr>
<tr>
<td><code>set_states(states[, prefix])</code></td>
<td>Set the number/names of the system states.</td>
</tr>
<tr>
<td><code>zero()</code></td>
<td>Compute the zeros of a state space system.</td>
</tr>
</tbody>
</table>

**append** *(other)*

Append a second model to the present model. The second model is converted to state-space if necessary, inputs and outputs are appended and their order is preserved.

**copy** ()

Make a copy of an input/output system.

**damp** ()

Natural frequency, damping ratio of system poles

**Returns**

- **wn** *(array)* – Natural frequencies for each system pole
- **zeta** *(array)* – Damping ratio for each system pole
- **poles** *(array)* – Array of system poles

**dcmgain** ()

Return the zero-frequency gain

The zero-frequency gain of a continuous-time state-space system is given by:

and of a discrete-time state-space system by:

**Returns** **gain** – An array of shape (outputs,inputs); the array will either be the zero-frequency (or DC) gain, or, if the frequency response is singular, the array will be filled with np.nan.

**Return type** ndarray

**evalfr** *(omega)*

Evaluate a SS system’s transfer function at a single frequency.

self._evalfr(omega) returns the value of the transfer function matrix with input value s = i * omega.

**feedback** *(other=1, sign=-1, params={})*

Feedback interconnection between two input/output systems

**Parameters**

- **sys1** *(InputOutputSystem)* – The primary process.
- **sys2** *(InputOutputSystem)* – The feedback process (often a feedback controller).
- **sign** *(scalar, optional)* – The sign of feedback. **sign** = -1 indicates negative feedback, and **sign** = 1 indicates positive feedback. **sign** is an optional argument; it assumes a value of -1 if not specified.

**Returns** **out**
Return type: *InputOutputSystem*

**Raises** *ValueError* – if the inputs, outputs, or timebases of the systems are incompatible.

**find_input** *(name)*
Find the index for an input given its name (*None* if not found)

**find_output** *(name)*
Find the index for an output given its name (*None* if not found)

**find_state** *(name)*
Find the index for a state given its name (*None* if not found)

**freqresp** *(omega)*
Evaluate the system’s transfer func. at a list of freqs, omega.

```python
mag, phase, omega = self.freqresp(omega)
```

Reports the frequency response of the system,

\[ G(j\omega) = \text{mag} \cdot \exp(j\text{phase}) \]

for continuous time. For discrete time systems, the response is evaluated around the unit circle such that

\[ G(\exp(j\omega dt)) = \text{mag} \cdot \exp(j\text{phase}). \]

**Parameters**
- **omega** *(array)* – A list of frequencies in radians/sec at which the system should be evaluated. The list can be either a python list or a numpy array and will be sorted before evaluation.

**Returns**
- **mag** *(float)* – The magnitude (absolute value, not dB or log10) of the system frequency response.
- **phase** *(float)* – The wrapped phase in radians of the system frequency response.
- **omega** *(array)* – The list of sorted frequencies at which the response was evaluated.

**horner** *(s)*
Evaluate the systems’s transfer function for a complex variable

Returns a matrix of values evaluated at complex variable s.

**isctime** *(strict=False)*
Check to see if a system is a continuous-time system

**Parameters**
- **sys** *(LTI system)* – System to be checked
- **strict** *(bool, optional)* – If strict is True, make sure that timebase is not None. Default is False.

**isdtime** *(strict=False)*
Check to see if a system is a discrete-time system

**Parameters**
- **strict** *(bool, optional)* – If strict is True, make sure that timebase is not None. Default is False.

**issiso** *
Check to see if a system is single input, single output
lft \( (\text{other}, \text{nu}=-1, \text{ny}=-1) \)
Return the Linear Fractional Transformation.

A definition of the LFT operator can be found in Appendix A.7, page 512 in the 2nd Edition, Multivariable Feedback Control by Sigurd Skogestad.

An alternative definition can be found here: https://www.mathworks.com/help/control/ref/lft.html

Parameters
- \text{other} (\text{LTI}) – The lower LTI system
- \text{ny} (\text{int}, \text{optional}) – Dimension of (plant) measurement output.
- \text{nu} (\text{int}, \text{optional}) – Dimension of (plant) control input.

linearize \( (x0, u0, t=0, \text{params}=\{\}, \text{eps}=1e-06) \)
Linearize an input/output system at a given state and input.

Return the linearization of an input/output system at a given state and input value as a StateSpace system. See \text{linearize()} for complete documentation.

minreal \( (\text{tol}=0.0) \)
Calculate a minimal realization, removes unobservable and uncontrollable states

pole()
Compute the poles of a state space system.

returnScipySignalLTI()
Return a list of a list of scipy.signal.lti objects.

For instance,

```python
>>> out = ssobject.returnScipySignalLTI()
>>> out[3][5]
```

is a signal.scipy.lti object corresponding to the transfer function from the 6th input to the 4th output.

sample \( (\text{Ts}, \text{method}='\text{zoh}', \alpha=\text{None}) \)
Convert a continuous time system to discrete time

Creates a discrete-time system from a continuous-time system by sampling. Multiple methods of conversion are supported.

Parameters
- \text{Ts} (\text{float}) – Sampling period
- \text{method} (\{"gbt", "bilinear", "euler", "backward_diff", "zoh"\})
  – Which method to use:
  - gbt: generalized bilinear transformation
  - bilinear: Tustin’s approximation ("gbt" with alpha=0.5)
  - euler: Euler (or forward differencing) method ("gbt" with alpha=0)
  - backward_diff: Backwards differencing ("gbt" with alpha=1.0)
  - zoh: zero-order hold (default)
- \alpha (\text{float within [0, 1]}) – The generalized bilinear transformation weighting parameter, which should only be specified with \text{method}="gbt", and is ignored otherwise

Returns \text{sysd} – Discrete time system, with sampling rate \text{Ts}
Return type  
*StateSpace*

Notes

Uses the command ‘cont2discrete’ from scipy.signal

Examples

```python
>>> sys = StateSpace(0, 1, 1, 0)
>>> sysd = sys.sample(0.5, method='bilinear')
```

**set_inputs**(inputs, prefix='u')

Set the number/names of the system inputs.

Parameters

- **inputs** *(int, list of str, or None)* – Description of the system inputs. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form u[i] (where the prefix u can be changed using the optional prefix parameter).

- **prefix** *(string, optional)* – If inputs is an integer, create the names of the states using the given prefix (default = ‘u’). The names of the input will be of the form prefix[i].

**set_outputs**(outputs, prefix='y')

Set the number/names of the system outputs.

Parameters

- **outputs** *(int, list of str, or None)* – Description of the system outputs. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form u[i] (where the prefix u can be changed using the optional prefix parameter).

- **prefix** *(string, optional)* – If outputs is an integer, create the names of the states using the given prefix (default = ‘u’). The names of the input will be of the form prefix[i].

**set_states**(states, prefix='x')

Set the number/names of the system states.

Parameters

- **states** *(int, list of str, or None)* – Description of the system states. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form u[i] (where the prefix u can be changed using the optional prefix parameter).

- **prefix** *(string, optional)* – If states is an integer, create the names of the states using the given prefix (default = ‘u’). The names of the input will be of the form prefix[i].

**zero**( )

Compute the zeros of a state space system.

4.5.2  control.NonlinearIOSystem

class control.NonlinearIOSystem(updfcn, outfcn=None, inputs=None, outputs=None, states=None, params={}, dt=None, name=None)

Nonlinear I/O system.
This class is used to implement a system that is a nonlinear state space system (defined by an update function and an output function).

```python
__init__(updfcn=None, outfcn=None, inputs=None, outputs=None, states=None, params={}, dt=None, name=None)
```

Create a nonlinear I/O system given update and output functions.

Creates an `InputOutputSystem` for a nonlinear system by specifying a state update function and an output function. The new system can be a continuous or discrete time system (Note: discrete-time systems not yet supported by most function.)

**Parameters**

- **updfcn** *(callable)* – Function returning the state update function
  ```python
  updfcn(t, x, u[, param]) -> array
  ```
  where `x` is a 1-D array with shape `(nstates,)`, `u` is a 1-D array with shape `(ninputs,)`, `t` is a float representing the current time, and `param` is an optional dict containing the values of parameters used by the function.

- **outfcn** *(callable)* – Function returning the output at the given state
  ```python
  outfcn(t, x, u[, param]) -> array
  ```
  where the arguments are the same as for `upfcn`.

- **inputs** *(int, list of str or None, optional)* – Description of the system inputs. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form `s[i]` (where `s` is one of `u`, `y`, or `x`). If this parameter is not given or given as `None`, the relevant quantity will be determined when possible based on other information provided to functions using the system.

- **outputs** *(int, list of str or None, optional)* – Description of the system outputs. Same format as `inputs`.

- **states** *(int, list of str, or None, optional)* – Description of the system states. Same format as `inputs`.

- **params** *(dict, optional)* – Parameter values for the systems. Passed to the evaluation functions for the system as default values, overriding internal defaults.

- **dt** *(timebase, optional)* – The timebase for the system, used to specify whether the system is operating in continuous or discrete time. It can have the following values:
  - `dt = None` No timebase specified
  - `dt = 0` Continuous time system
  - `dt > 0` Discrete time system with sampling time `dt`
  - `dt = True` Discrete time with unspecified sampling time

- **name** *(string, optional)* – System name (used for specifying signals).

**Returns** `iosys` – Nonlinear system represented as an input/output system.

**Return type** `NonlinearIOSystem`

## Methods

### 4.5. Input/Output system subclasses
__init__(updfcn[, outfcn, inputs, outputs, ...])
Create a nonlinear I/O system given update and output functions.

copy()
Make a copy of an input/output system.

feedback([other, sign, params])
Feedback interconnection between two input/output systems

find_input(name)
Find the index for an input given its name (None if not found)

find_output(name)
Find the index for an output given its name (None if not found)

find_state(name)
Find the index for a state given its name (None if not found)

linearize(x0, u0[, t, params, eps])
Linearize an input/output system at a given state and input.

set_inputs(inputs[, prefix])
Set the number/names of the system inputs.

set_outputs(outputs[, prefix])
Set the number/names of the system outputs.

set_states(states[, prefix])
Set the number/names of the system states.

copy()
Make a copy of an input/output system.

feedback(other=1, sign=-1, params={})
Feedback interconnection between two input/output systems

Parameters

- sys1 (InputOutputSystem) – The primary process.
- sys2 (InputOutputSystem) – The feedback process (often a feedback controller).
- sign (scalar, optional) – The sign of feedback. sign = -1 indicates negative feedback, and sign = 1 indicates positive feedback. sign is an optional argument; it assumes a value of -1 if not specified.

Returns out

Return type InputOutputSystem

Raises ValueError – if the inputs, outputs, or timebases of the systems are incompatible.

find_input(name)
Find the index for an input given its name (None if not found)

find_output(name)
Find the index for an output given its name (None if not found)

find_state(name)
Find the index for a state given its name (None if not found)

linearize(x0, u0, t=0, params={}, eps=1e-06)
Linearize an input/output system at a given state and input.

Return the linearization of an input/output system at a given state and input value as a StateSpace system. See linearize() for complete documentation.

set_inputs(inputs, prefix='u')
Set the number/names of the system inputs.

Parameters
• **inputs** (*int, list of str, or None*) – Description of the system inputs. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form $u[i]$ (where the prefix $u$ can be changed using the optional prefix parameter).

• **prefix** (*string, optional*) – If `inputs` is an integer, create the names of the states using the given prefix (default = ‘u’). The names of the input will be of the form `prefix[i]`.

`set_outputs`(outputs, prefix='y')

Set the number/names of the system outputs.

Parameters

• **outputs** (*int, list of str, or None*) – Description of the system outputs. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form $u[i]$ (where the prefix $u$ can be changed using the optional prefix parameter).

• **prefix** (*string, optional*) – If `outputs` is an integer, create the names of the states using the given prefix (default = ‘u’). The names of the input will be of the form `prefix[i]`.

`set_states`(states, prefix='x')

Set the number/names of the system states.

Parameters

• **states** (*int, list of str, or None*) – Description of the system states. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form $u[i]$ (where the prefix $u$ can be changed using the optional prefix parameter).

• **prefix** (*string, optional*) – If `states` is an integer, create the names of the states using the given prefix (default = ‘u’). The names of the input will be of the form `prefix[i]`.

### 4.5.3 `control.InterconnectedSystem`

class `control.InterconnectedSystem`(syslist, connections=[], inplist=[], outlist=[], inputs=None, outputs=None, states=None, params={}, dt=None, name=None)

Interconnection of a set of input/output systems.

This class is used to implement a system that is an interconnection of input/output systems. The `sys` consists of a collection of subsystems whose inputs and outputs are connected via a connection map. The overall system inputs and outputs are subsets of the subsystem inputs and outputs.

```
__init__(syslist, connections=[], inplist=[], outlist=[], inputs=None, outputs=None, states=None, params={}, dt=None, name=None)
```

Create an I/O system from a list of systems + connection info.

The `InterconnectedSystem` class is used to represent an input/output system that consists of an interconnection between a set of subsystems. The outputs of each subsystem can be summed together to provide inputs to other subsystems. The overall system inputs and outputs can be any subset of subsystem inputs and outputs.

Parameters

• **syslist** (*array_like of InputOutputSystems*) – The list of input/output systems to be connected
• **connections** *(tuple of connection specifications, optional)* – Description of the internal connections between the subsystems. Each element of the tuple describes an input to one of the subsystems. The entries are of the form:

  \[(\text{input-spec, output-spec}_1, \text{output-spec}_2, \ldots)\]

  The input-spec should be a tuple of the form \((\text{subsys}_i, \text{inp}_j)\) where \text{subsys}_i is the index into \text{syslist} and \text{inp}_j is the index into the input vector for the subsystem. If \text{subsys}_i has a single input, then the subsystem index \text{subsys}_i can be listed as the input-spec. If systems and signals are given names, then the form ‘sys.sig’ or (‘sys’, ‘sig’) are also recognized.

  Each output-spec should be a tuple of the form \((\text{subsys}_i, \text{out}_j, \text{gain})\). The input will be constructed by summing the listed outputs after multiplying by the gain term. If the gain term is omitted, it is assumed to be 1. If the system has a single output, then the subsystem index \text{subsys}_i can be listed as the input-spec. If systems and signals are given names, then the form ‘sys.sig’, (‘sys’, ‘sig’) or (‘sys’, ‘sig’, gain) are also recognized, and the special form ‘-sys.sig’ can be used to specify a signal with gain -1.

  If omitted, the connection map (matrix) can be specified using the `set_connect_map()` method.

• **inplist** *(tuple of input specifications, optional)* – List of specifications for how the inputs for the overall system are mapped to the subsystems. The input specification is the same as the form defined in the connection specification. Each system input is added to the input for the listed subsystem.

  If omitted, the input map can be specified using the `set_input_map` method.

• **outlist** *(tuple of output specifications, optional)* – List of specifications for how the outputs for the subsystems are mapped to overall system. The output specification is the same as the form defined in the connection specification (including the optional gain term). Numbered outputs must be chosen from the list of subsystem outputs, but named outputs can also be contained in the list of subsystem inputs.

  If omitted, the output map can be specified using the `set_output_map` method.

• **params** *(dict, optional)* – Parameter values for the systems. Passed to the evaluation functions for the system as default values, overriding internal defaults.

• **dt** *(timebase, optional)* – The timebase for the system, used to specify whether the system is operating in continuous or discrete time. It can have the following values:

  - `dt = None` No timebase specified
  - `dt = 0` Continuous time system
  - `dt > 0` Discrete time system with sampling time `dt`
  - `dt = True` Discrete time with unspecified sampling time

• **name** *(string, optional)* – System name (used for specifying signals).

## Methods

<table>
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<tr>
<th>Method</th>
<th>Description</th>
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<td>Create an I/O system from a list of systems + connection info.</td>
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<tr>
<td><code>copy()</code></td>
<td>Make a copy of an input/output system.</td>
</tr>
<tr>
<td><code>feedback([other, sign, params])</code></td>
<td>Feedback interconnection between two input/output systems</td>
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<td>find_input(name)</td>
<td>Find the index for an input given its name (None if not found)</td>
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<tr>
<td>find_output(name)</td>
<td>Find the index for an output given its name (None if not found)</td>
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<tr>
<td>find_state(name)</td>
<td>Find the index for a state given its name (None if not found)</td>
</tr>
<tr>
<td>linearize(x0, u0[, t, params, eps])</td>
<td>Linearize an input/output system at a given state and input.</td>
</tr>
<tr>
<td>set_connect_map(connect_map)</td>
<td>Set the connection map for an interconnected I/O system.</td>
</tr>
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</tr>
<tr>
<td>set_output_map(output_map)</td>
<td>Set the output map for an interconnected I/O system.</td>
</tr>
<tr>
<td>set_outputs(outputs[, prefix])</td>
<td>Set the number/names of the system outputs.</td>
</tr>
<tr>
<td>set_states(states[, prefix])</td>
<td>Set the number/names of the system states.</td>
</tr>
</tbody>
</table>

**copy()**

Make a copy of an input/output system.

**feedback(other=1, sign=-1, params=())**

Feedback interconnection between two input/output systems

**Parameters**

- **sys1 (InputOutputSystem)** – The primary process.
- **sys2 (InputOutputSystem)** – The feedback process (often a feedback controller).
- **sign (scalar, optional)** – The sign of feedback. \( sign = -1 \) indicates negative feedback, and \( sign = 1 \) indicates positive feedback. \( sign \) is an optional argument; it assumes a value of \(-1\) if not specified.

**Returns out**

**Return type** InputOutputSystem

**Raises** ValueError – if the inputs, outputs, or timebases of the systems are incompatible.

**find_input(name)**

Find the index for an input given its name (None if not found)

**find_output(name)**

Find the index for an output given its name (None if not found)

**find_state(name)**

Find the index for a state given its name (None if not found)

**linearize(x0, u0[, t, params, eps])**

Linearize an input/output system at a given state and input.

Return the linearization of an input/output system at a given state and input value as a StateSpace system. See linearize() for complete documentation.

**set_connect_map(connect_map)**

Set the connection map for an interconnected I/O system.

**Parameters**

- **connect_map (2D array)** – Specify the matrix that will be used to multiply the vector of subsystem outputs to obtain the vector of subsystem inputs.
**set_input_map** (*input_map*)

Set the input map for an interconnected I/O system.

**Parameters**

- **input_map** (*2D array*) – Specify the matrix that will be used to multiply the vector of system inputs to obtain the vector of subsystem inputs. These values are added to the inputs specified in the connection map.

**set_inputs** (*inputs, prefix='u'*)

Set the number/names of the system inputs.

**Parameters**

- **inputs** (*int, list of str, or None*) – Description of the system inputs. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form $u[i]$ (where the prefix $u$ can be changed using the optional prefix parameter).

- **prefix** (*string, optional*) – If *inputs* is an integer, create the names of the states using the given prefix (default = ‘u’). The names of the input will be of the form $prefix[i]$.

**set_output_map** (*output_map*)

Set the output map for an interconnected I/O system.

**Parameters**

- **output_map** (*2D array*) – Specify the matrix that will be used to multiply the vector of subsystem outputs to obtain the vector of system outputs.

**set_outputs** (*outputs, prefix='y'*)

Set the number/names of the system outputs.

**Parameters**

- **outputs** (*int, list of str, or None*) – Description of the system outputs. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form $u[i]$ (where the prefix $u$ can be changed using the optional prefix parameter).

- **prefix** (*string, optional*) – If *outputs* is an integer, create the names of the states using the given prefix (default = ‘u’). The names of the input will be of the form $prefix[i]$.

**set_states** (*states, prefix='x'*)

Set the number/names of the system states.

**Parameters**

- **states** (*int, list of str, or None*) – Description of the system states. This can be given as an integer count or as a list of strings that name the individual signals. If an integer count is specified, the names of the signal will be of the form $u[i]$ (where the prefix $u$ can be changed using the optional prefix parameter).

- **prefix** (*string, optional*) – If *states* is an integer, create the names of the states using the given prefix (default = ‘u’). The names of the input will be of the form $prefix[i]$. 
The `control.matlab` module contains a number of functions that emulate some of the functionality of MATLAB. The intent of these functions is to provide a simple interface to the python control systems library (python-control) for people who are familiar with the MATLAB Control Systems Toolbox (tm).

## 5.1 Creating linear models

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<td><code>tf(num, den[, dt])</code></td>
<td>Create a transfer function system. Can create MIMO systems.</td>
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<td><code>ss(A, B, C, D[, dt])</code></td>
<td>Create a state space system.</td>
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<td><code>frd(d, w)</code></td>
<td>Construct a frequency response data model</td>
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<tr>
<td><code>rss([states, outputs, inputs])</code></td>
<td>Create a stable continuous random state space object.</td>
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<tr>
<td><code>drss([states, outputs, inputs])</code></td>
<td>Create a stable discrete random state space object.</td>
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</tbody>
</table>

### 5.1.1 control.matlab.tf

`control.matlab.tf(num, den[, dt])`

Create a transfer function system. Can create MIMO systems.

The function accepts either 1, 2, or 3 parameters:

- `tf(sys)` Convert a linear system into transfer function form. Always creates a new system, even if sys is already a TransferFunction object.
- `tf(num, den)` Create a transfer function system from its numerator and denominator polynomial coefficients.
  - If `num` and `den` are 1D array_like objects, the function creates a SISO system.
  - To create a MIMO system, `num` and `den` need to be 2D nested lists of array_like objects. (A 3 dimensional data structure in total.) (For details see note below.)
- `tf(num, den, dt)` Create a discrete time transfer function system; `dt` can either be a positive number indicating the sampling time or ‘True’ if no specific timebase is given.
tf('s') or tf('z') Create a transfer function representing the differential operator ('s') or delay operator ('z').

Parameters
• sys (LTI (StateSpace or TransferFunction)) – A linear system
• num (array_like, or list of list of array_like) – Polynomial coefficients of the numerator
• den (array_like, or list of list of array_like) – Polynomial coefficients of the denominator

Returns out – The new linear system

Return type TransferFunction

Raises
• ValueError – if num and den have invalid or unequal dimensions
• TypeError – if num or den are of incorrect type

See also:
TransferFunction(), ss(), ss2tf(), tf2ss()

Notes
num[i][j] contains the polynomial coefficients of the numerator for the transfer function from the (j+1)st input to the (i+1)st output. den[i][j] works the same way.
The list [2, 3, 4] denotes the polynomial $2s^2 + 3s + 4$.
The special forms tf('s') and tf('z') can be used to create transfer functions for differentiation and unit delays.

Examples

```python
>>> # Create a MIMO transfer function object
>>> # The transfer function from the 2nd input to the 1st output is
>>> # (3s + 4) / (6s^2 + 5s + 4).
>>> num = [[[1., 2.], [3., 4.]], [[5., 6.], [7., 8.]]]
>>> den = [[[9., 8., 7.], [6., 5., 4.]], [[3., 2., 1.], [-1., -2., -3.]]]
>>> sys1 = tf(num, den)

>>> # Create a variable 's' to allow algebra operations for SISO systems
>>> s = tf('s')
>>> G = (s + 1)/(s**2 + 2*s + 1)

>>> # Convert a StateSpace to a TransferFunction object.
>>> sys_ss = ss("1. -2; 3. -4", "5.; 7", "6. 8", "9."
>>> sys2 = tf(sys1)
```
control.matlab.ss

control.matlab.ss(A, B, C, D[, dt])

Create a state space system.

The function accepts either 1, 4 or 5 parameters:

- **ss(sys)**: Convert a linear system into state space form. Always creates a new system, even if sys is already a StateSpace object.

- **ss(A, B, C, D)**: Create a state space system from the matrices of its state and output equations:
  \[
  \dot{x} = A \cdot x + B \cdot u \\
  y = C \cdot x + D \cdot u
  \]

- **ss(A, B, C, D, dt)**: Create a discrete-time state space system from the matrices of its state and output equations:
  \[
  x[k+1] = A \cdot x[k] + B \cdot u[k] \\
  y[k] = C \cdot x[k] + D \cdot u[k]
  \]

The matrices can be given as *array like* data types or strings. Everything that the constructor of *numpy.matrix* accepts is permissible here too.

**Parameters**

- **sys** (*StateSpace* or *TransferFunction*) – A linear system
- **A** (*array_like or string*) – System matrix
- **B** (*array_like or string*) – Control matrix
- **C** (*array_like or string*) – Output matrix
- **D** (*array_like or string*) – Feed forward matrix
- **dt** (*If present, specifies the sampling period and a discrete time*) – system is created

**Returns** out – The new linear system

**Return type** *StateSpace*

**Raises** *ValueError* – if matrix sizes are not self-consistent

**See also:**
*StateSpace(), tf(), ss2tf(), tf2ss()*

**Examples**

```python
>>> # Create a StateSpace object from four "matrices".
>>> sys1 = ss("1. -2; 3. -4", "5.; 7", "6. 8", "9.")
```

```python
>>> # Convert a TransferFunction to a StateSpace object.
>>> sys_tf = tf([2.], [1., 3])
>>> sys2 = ss(sys_tf)
```
5.1.3 control.matlab.fr

control.matlab.fr(d, w)
Construct a frequency response data model
frd models store the (measured) frequency response of a system.
This function can be called in different ways:
frd(response, freqs) Create an frd model with the given response data, in the form of complex response vector, at matching frequency freqs [in rad/s]
frd(sys, freqs) Convert an LTI system into an frd model with data at frequencies freqs.

Parameters
• response(array_like, or list) – complex vector with the system response
• freq(array_like or list) – vector with frequencies
• sys(LTI (StateSpace or TransferFunction)) – A linear system

Returns sys – New frequency response system
Return type FRD

See also:
FRD(), ss(), tf()

5.1.4 control.matlabrss

control.matlabrss(states=1, outputs=1, inputs=1)
Create a stable continuous random state space object.

Parameters
• states(integer) – Number of state variables
• inputs(integer) – Number of system inputs
• outputs(integer) – Number of system outputs

Returns sys – The randomly created linear system
Return type StateSpace

Raises ValueError – if any input is not a positive integer

See also:
drss()

Notes
If the number of states, inputs, or outputs is not specified, then the missing numbers are assumed to be 1. The poles of the returned system will always have a negative real part.
5.1.5 control.matlab.drss

control.matlab.drss(states=1, outputs=1, inputs=1)
Create a stable discrete random state space object.

Parameters

• states (integer) – Number of state variables
• inputs (integer) – Number of system inputs
• outputs (integer) – Number of system outputs

Returns sys – The randomly created linear system

Return type StateSpace

Raises ValueError – if any input is not a positive integer

See also:

rss()

Notes

If the number of states, inputs, or outputs is not specified, then the missing numbers are assumed to be 1. The poles of the returned system will always have a magnitude less than 1.

5.2 Utility functions and conversions

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<td>Convert a magnitude to decibels (dB)</td>
</tr>
<tr>
<td>db2mag(db)</td>
<td>Convert a gain in decibels (dB) to a magnitude</td>
</tr>
<tr>
<td>c2d(sysc, Ts[, method])</td>
<td>Return a discrete-time system</td>
</tr>
<tr>
<td>ss2tf(sys)</td>
<td>Transform a state space system to a transfer function.</td>
</tr>
<tr>
<td>tf2ss(sys)</td>
<td>Transform a transfer function to a state space system.</td>
</tr>
<tr>
<td>tfdata(sys)</td>
<td>Return transfer function data objects for a system</td>
</tr>
</tbody>
</table>

5.2.1 control.matlab.mag2db

control.matlab.mag2db(mag)
Convert a magnitude to decibels (dB)

If A is magnitude,

\[ db = 20 \times \log_{10}(A) \]

Parameters mag (float or ndarray) – input magnitude or array of magnitudes

Returns db – corresponding values in decibels

Return type float or ndarray

5.2.2 control.matlab.db2mag

control.matlab.db2mag(db)
Convert a gain in decibels (dB) to a magnitude
If $A$ is magnitude,
\[ db = 20 \times \log_{10}(A) \]

**Parameters**
- \texttt{db} (*float or ndarray*) – input value or array of values, given in decibels

**Returns**
- \texttt{mag} – corresponding magnitudes

**Return type**
- \texttt{float or ndarray}

### 5.2.3 control.matlab.c2d

\texttt{control.matlab.c2d}(sysc, Ts, method='zoh')

Return a discrete-time system

**Parameters**
- \texttt{sysc} (*LTI (StateSpace or TransferFunction), continuous*) – System to be converted
- \texttt{Ts} (*number*) – Sample time for the conversion
- \texttt{method} (*string, optional*) – Method to be applied, ’zoh’ Zero-order hold on the inputs (default) ’foh’ First-order hold, currently not implemented ’impulse’ Impulse-invariant discretization, currently not implemented ’tustin’ Bilinear (Tustin) approximation, only SISO ’matched’ Matched pole-zero method, only SISO

### 5.2.4 control.matlab.ss2tf

\texttt{control.matlab.ss2tf}(sys)

Transform a state space system to a transfer function.

The function accepts either 1 or 4 parameters:

- \texttt{ss2tf(sys)} Convert a linear system into space system form. Always creates a new system, even if sys is already a StateSpace object.
- \texttt{ss2tf(A, B, C, D)} Create a state space system from the matrices of its state and output equations.

For details see: \texttt{ss()}

**Parameters**
- \texttt{sys} (*StateSpace*) – A linear system
- \texttt{A} (*array_like or string*) – System matrix
- \texttt{B} (*array_like or string*) – Control matrix
- \texttt{C} (*array_like or string*) – Output matrix
- \texttt{D} (*array_like or string*) – Feedthrough matrix

**Returns**
- \texttt{out} – New linear system in transfer function form

**Return type** \texttt{TransferFunction}

**Raises**
- \texttt{ValueError} – if matrix sizes are not self-consistent, or if an invalid number of arguments is passed in
- \texttt{TypeError} – if \texttt{sys} is not a StateSpace object

---

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See also:

`tf()`, `ss()`, `tf2ss()`

**Examples**

```python
>>> A = [[1., -2], [3, -4]]
>>> B = [[5.], [7]]
>>> C = [[6., 8]]
>>> D = [[9.]]
>>> sys1 = ss2tf(A, B, C, D)
>>> sys_ss = ss(A, B, C, D)
>>> sys2 = ss2tf(sys_ss)
```

### 5.2.5 `control.matlab.tf2ss`

`control.matlab.tf2ss(sys)`

Transform a transfer function to a state space system.

The function accepts either 1 or 2 parameters:

- `tf2ss(sys)` Convert a linear system into transfer function form. Always creates a new system, even if `sys` is already a TransferFunction object.

- `tf2ss(num, den)` Create a transfer function system from its numerator and denominator polynomial coefficients.

For details see: `tf()`

**Parameters**

- `sys` (*LTI (StateSpace or TransferFunction)*) – A linear system
- `num` (*array_like, or list of list of array_like*) – Polynomial coefficients of the numerator
- `den` (*array_like, or list of list of array_like*) – Polynomial coefficients of the denominator

**Returns** `out` – New linear system in state space form

**Return type** `StateSpace`

**Raises**

- `ValueError` – if `num` and `den` have invalid or unequal dimensions, or if an invalid number of arguments is passed in
- `TypeError` – if `num` or `den` are of incorrect type, or if `sys` is not a TransferFunction object

**See also:**

`sstf()`, `tf()`, `ss2tf()`
Examples

```
>>> num = [[[1., 2.], [3., 4.]], [[5., 6.], [7., 8.]]]
>>> den = [[[9., 8., 7.], [6., 5., 4.]], [[[3., 2., 1.], [-1., -2., -3.]]]
>>> sys1 = tf2ss(num, den)
>>> sys_tf = tf(num, den)
>>> sys2 = tf2ss(sys_tf)
```

5.2.6 control.matlab.tfdata

`control.matlab.tfdata(sys)`

Return transfer function data objects for a system

**Parameters** `sys (LTI (StateSpace, or TransferFunction))` – LTI system whose data will be returned

**Returns** `(num, den)` – Transfer function coefficients (SISO only)

**Return type** numerator and denominator arrays

5.3 System interconnections

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<th>Description</th>
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</thead>
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<td><code>series(sys1,*sysn)</code></td>
<td>Return the series connection (... * sys3 *) sys2 * sys1</td>
</tr>
<tr>
<td><code>parallel(sys1,*sysn)</code></td>
<td>Return the parallel connection sys1 + sys2 (+ sys3 + ...)</td>
</tr>
<tr>
<td><code>feedback(sys1[,sys2,sign])</code></td>
<td>Feedback interconnection between two I/O systems.</td>
</tr>
<tr>
<td><code>negate(sys)</code></td>
<td>Return the negative of a system.</td>
</tr>
<tr>
<td><code>connect(sys,Q,inputv,outputv)</code></td>
<td>Index-base interconnection of system</td>
</tr>
<tr>
<td><code>append(sys1,sys2,...,sysn)</code></td>
<td>Group models by appending their inputs and outputs</td>
</tr>
</tbody>
</table>

5.3.1 control.matlab.series

`control.matlab.series(sys1,*sysn)`

Return the series connection (... * sys3 *) sys2 * sys1

**Parameters**

- `sys1 (scalar, StateSpace, TransferFunction, or FRD)` -
- `sysn (other scalars, StateSpaces, TransferFunctions, or FRDs)` -

**Returns** `out`

**Return type** scalar, `StateSpace`, or `TransferFunction`

**Raises** `ValueError` – if `sys2.inputs` does not equal `sys1.outputs` if `sys1.dt` is not compatible with `sys2.dt`

See also: `parallel()`, `feedback()`
Notes

This function is a wrapper for the __mul__ function in the StateSpace and TransferFunction classes. The output type is usually the type of sys2. If sys2 is a scalar, then the output type is the type of sys1.

If both systems have a defined timebase (dt = 0 for continuous time, dt > 0 for discrete time), then the timebase for both systems must match. If only one of the system has a timebase, the return timebase will be set to match it.

Examples

```python
>>> sys3 = series(sys1, sys2)  # Same as sys3 = sys2 * sys1

>>> sys5 = series(sys1, sys2, sys3, sys4)  # More systems
```

5.3.2 control.matlab.parallel

control.matlab.parallel(sys1, *sysn)

Return the parallel connection sys1 + sys2 (+ sys3 + ...)

Parameters

• `sys1 (scalar, StateSpace, TransferFunction, or FRD)` -
• `*sysn (other scalars, StateSpaces, TransferFunctions, or FRDs)` -

Returns out

Return type  scalar, StateSpace, or TransferFunction

Raises ValueError – if `sys1` and `sys2` do not have the same numbers of inputs and outputs

See also:

`series()`, `feedback()`

Notes

This function is a wrapper for the __add__ function in the StateSpace and TransferFunction classes. The output type is usually the type of sys1. If sys1 is a scalar, then the output type is the type of sys2.

If both systems have a defined timebase (dt = 0 for continuous time, dt > 0 for discrete time), then the timebase for both systems must match. If only one of the system has a timebase, the return timebase will be set to match it.

Examples

```python
>>> sys3 = parallel(sys1, sys2)  # Same as sys3 = sys1 + sys2

>>> sys5 = parallel(sys1, sys2, sys3, sys4)  # More systems
```
5.3.3 control.matlab.feedback

control.matlab.feedback(sys1, sys2=1, sign=-1)
Feedback interconnection between two I/O systems.

Parameters

- `sys1` (scalar, StateSpace, TransferFunction, FRD) – The primary plant.
- `sys2` (scalar, StateSpace, TransferFunction, FRD) – The feedback plant (often a feedback controller).
- `sign` (scalar) – The sign of feedback. `sign = -1` indicates negative feedback, and `sign = 1` indicates positive feedback. `sign` is an optional argument; it assumes a value of -1 if not specified.

Returns out

Return type `StateSpace` or `TransferFunction`

Raises

- `ValueError` – if `sys1` does not have as many inputs as `sys2` has outputs, or if `sys2` does not have as many inputs as `sys1` has outputs
- `NotImplementedError` – if an attempt is made to perform a feedback on a MIMO TransferFunction object

See also:

`series()`, `parallel()`

Notes

This function is a wrapper for the feedback function in the StateSpace and TransferFunction classes. It calls `TransferFunction.feedback` if `sys1` is a TransferFunction object, and `StateSpace.feedback` if `sys1` is a StateSpace object. If `sys1` is a scalar, then it is converted to `sys2`'s type, and the corresponding feedback function is used. If `sys1` and `sys2` are both scalars, then `TransferFunction.feedback` is used.

5.3.4 control.matlab.negate

control.matlab.negate(sys)
Return the negative of a system.

Parameters `sys` (StateSpace, TransferFunction or FRD) –

Returns out

Return type `StateSpace` or `TransferFunction`

Notes

This function is a wrapper for the `__neg__` function in the StateSpace and TransferFunction classes. The output type is the same as the input type.

If both systems have a defined timebase (dt = 0 for continuous time, dt > 0 for discrete time), then the timebase for both systems must match. If only one of the system has a timebase, the return timebase will be set to match it.
Examples

```python
>>> sys2 = negate(sys1) # Same as sys2 = -sys1.
```

### 5.3.5 control.matlab.connect

**control.matlab.connect** *(sys, Q, inputv, outputv)*

Index-base interconnection of system

The system sys is a system typically constructed with append, with multiple inputs and outputs. The inputs and outputs are connected according to the interconnection matrix Q, and then the final inputs and outputs are trimmed according to the inputs and outputs listed in inputv and outputv.

Note: to have this work, inputs and outputs start counting at 1!!!!

**Parameters**

- **sys** *(StateSpace, Transferfunction)* – System to be connected
- **Q** *(2d array)* – Interconnection matrix. First column gives the input to be connected, second column gives the output to be fed into this input. Negative values for the second column mean the feedback is negative, 0 means no connection is made
- **inputv** *(1d array)* – list of final external inputs
- **outputv** *(1d array)* – list of final external outputs

**Returns** sys – Connected and trimmed LTI system

**Return type** LTI system

Examples

```python
>>> sys1 = ss("1. -2; 3. -4", "5.; 7", "6, 8", "9.")
>>> sys2 = ss("-1.", "1.", "1.", "0.")
>>> sys = append(sys1, sys2)
>>> Q = sp.mat([[ 1, 2], [2, -1]]) # basically feedback, output 2 in 1
>>> sysc = connect(sys, Q, [2], [1, 2])
```

### 5.3.6 control.matlab.append

**control.matlab.append** *(sys1, sys2, ..., sysn)*

Group models by appending their inputs and outputs

Forms an augmented system model, and appends the inputs and outputs together. The system type will be the type of the first system given; if you mix state-space systems and gain matrices, make sure the gain matrices are not first.

**Parameters** sys2, .. sysn (sys1, ) – LTI systems to combine

**Returns** sys – Combined LTI system, with input/output vectors consisting of all input/output vectors appended

**Return type** LTI system
Examples

```python
>>> sys1 = ss("1. -2; 3. -4", "5.; 7", "6. 8", "9."
>>> sys2 = ss("-1.", "1.", "1.", "0.")
>>> sys = append(sys1, sys2)
```

Todo: also implement for transfer function, zpk, etc.

## 5.4 System gain and dynamics

<table>
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<th>Function</th>
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<td><code>dcgain(*args)</code></td>
<td>Compute the gain of the system in steady state.</td>
</tr>
<tr>
<td><code>pole(sys)</code></td>
<td>Compute system poles.</td>
</tr>
<tr>
<td><code>zero(sys)</code></td>
<td>Compute system zeros.</td>
</tr>
<tr>
<td><code>damp(sys[, doprint])</code></td>
<td>Compute natural frequency, damping ratio, and poles of a system</td>
</tr>
<tr>
<td><code>pzmap(sys[, Plot, grid, title])</code></td>
<td>Plot a pole/zero map for a linear system.</td>
</tr>
</tbody>
</table>

### 5.4.1 control.matlab.dcgain

**control.matlab.dcgain(*args)**

Compute the gain of the system in steady state.

The function takes either 1, 2, 3, or 4 parameters:

**Parameters**

- `B, C, D (A,)` – A linear system in state space form.
- `P, k (Z,)` – A linear system in zero, pole, gain form.
- `den (num,)` – A linear system in transfer function form.
- `sys (LTI (StateSpace or TransferFunction))` – A linear system object.

**Returns**

- `gain` – The gain of each output versus each input: \( y = gain \cdot u \)

**Return type**

- ndarray

**Notes**

This function is only useful for systems with invertible system matrix \( A \).

All systems are first converted to state space form. The function then computes:

\[
gain = -C \cdot A^{-1} \cdot B + D
\]

### 5.4.2 control.matlab.pole

**control.matlab.pole(sys)**

Compute system poles.

**Parameters**

- `sys (StateSpace or TransferFunction)` – Linear system
Returns : **poles** – Array that contains the system’s poles.

Return type : ndarray

Raises : **NotImplementedError** – when called on a TransferFunction object

See also:

- `zero()`,
- `TransferFunction.pole()`,
- `StateSpace.pole()`

### 5.4.3 control.matlab.zero

```python
def control.matlab.zero(sys):
    Compute system zeros.
```

**Parameters**

- **sys** *(StateSpace or TransferFunction)* – Linear system

**Returns**

- **zeros** – Array that contains the system’s zeros.

**Return type** : ndarray

**Raises**

- **NotImplementedError** – when called on a MIMO system

See also:

- `pole()`,
- `StateSpace.zero()`,
- `TransferFunction.zero()`

### 5.4.4 control.matlab.damp

```python
def control.matlab.damp(sys, doprint=True):
    Compute natural frequency, damping ratio, and poles of a system
```

The function takes 1 or 2 parameters

**Parameters**

- **sys** *(LTI (StateSpace or TransferFunction))* – A linear system object
- **doprint** – if true, print table with values

**Returns**

- **wn** *(array)* – Natural frequencies of the poles
- **damping** *(array)* – Damping values
- **poles** *(array)* – Pole locations
- **Algorithm**
  - If the system is continuous, \( wn = \text{abs}(\text{poles}) \ Z = -\text{real}(\text{poles})/\text{poles}. \)
  - If the system is discrete, the discrete poles are mapped to their equivalent location in the s-plane via \( s = \text{log10}(\text{poles})/\text{dt} \) and \( wn = \text{abs}(s) \ Z = -\text{real}(s)/wn. \)

See also:

- `pole()`
5.4.5 control.matlab.pzmap

control.matlab.pzmap(sys, Plot=True, grid=False, title='Pole Zero Map')
Plot a pole/zero map for a linear system.

Parameters

- **sys** (*LTI (StateSpace or TransferFunction)*) – Linear system for which poles and zeros are computed.
- **Plot** (*bool*) – If True a graph is generated with Matplotlib, otherwise the poles and zeros are only computed and returned.
- **grid** (*boolean (default = False)*) – If True plot omega-damping grid.

Returns

- **pole** (*array*) – The system's poles
- **zeros** (*array*) – The system’s zeros.

5.5 Time-domain analysis

<table>
<thead>
<tr>
<th>function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>step(sys, T, X0, input, output, return_x)</td>
<td>Step response of a linear system</td>
</tr>
<tr>
<td>impulse(sys, T, X0, input, output, return_x)</td>
<td>Impulse response of a linear system</td>
</tr>
<tr>
<td>initial(sys, T, X0, input, output, return_x)</td>
<td>Initial condition response of a linear system</td>
</tr>
<tr>
<td>lsim(sys, U, T, X0)</td>
<td>Simulate the output of a linear system.</td>
</tr>
</tbody>
</table>

5.5.1 control.matlab.step

control.matlab.step(sys, T=None, X0=0.0, input=0, output=None, return_x=False)
Step response of a linear system

If the system has multiple inputs or outputs (MIMO), one input has to be selected for the simulation. Optionally, one output may be selected. If no selection is made for the output, all outputs are given. The parameters input and output do this. All other inputs are set to 0, all other outputs are ignored.

Parameters

- **sys** (*StateSpace, or TransferFunction*) – LTI system to simulate
- **T** (*array-like object, optional*) – Time vector (argument is autocomputed if not given)
- **X0** (*array-like or number, optional*) – Initial condition (default = 0)
  Numbers are converted to constant arrays with the correct shape.
- **input** (*int*) – Index of the input that will be used in this simulation.
- **output** (*int*) – If given, index of the output that is returned by this simulation.

Returns

- **yout** (*array*) – Response of the system
- **T** (*array*) – Time values of the output
- **xout** (*array (if selected)*) – Individual response of each x variable
See also:
lsim(), initial(), impulse()

Examples

```python
>>> yout, T = step(sys, T, X0)
```

5.5.2 control.matlab.impulse

control.matlab.impulse(sys, T=None, X0=0.0, input=0, output=None, return_x=False)
Impulse response of a linear system
If the system has multiple inputs or outputs (MIMO), one input has to be selected for the simulation. Optionally, one output may be selected. If no selection is made for the output, all outputs are given. The parameters input and output do this. All other inputs are set to 0, all other outputs are ignored.

Parameters
- **sys** (StateSpace, TransferFunction) – LTI system to simulate
- **T** (array-like object, optional) – Time vector (argument is autocomputed if not given)
- **X0** (array-like or number, optional) – Initial condition (default = 0)
  Numbers are converted to constant arrays with the correct shape.
- **input** (int) – Index of the input that will be used in this simulation.
- **output** (int) – Index of the output that will be used in this simulation.

Returns
- **yout** (array) – Response of the system
- **T** (array) – Time values of the output
- **xout** (array (if selected)) – Individual response of each x variable

See also:
lsim(), step(), initial()

Examples

```python
>>> yout, T = impulse(sys, T)
```

5.5.3 control.matlab.initial

control.matlab.initial(sys, T=None, X0=0.0, input=None, output=None, return_x=False)
Initial condition response of a linear system
If the system has multiple outputs (MIMO), optionally, one output may be selected. If no selection is made for the output, all outputs are given.

Parameters


- **sys** *(StateSpace, or TransferFunction)* – LTI system to simulate
- **T** *(array-like object, optional)* – Time vector (argument is autocomputed if not given)
- **X0** *(array-like object or number, optional)* – Initial condition (default = 0)

Numbers are converted to constant arrays with the correct shape.

- **input** *(int)* – This input is ignored, but present for compatibility with step and impulse.
- **output** *(int)* – If given, index of the output that is returned by this simulation.

**Returns**

- **yout** *(array)* – Response of the system
- **T** *(array)* – Time values of the output
- **xout** *(array (if selected))* – Individual response of each x variable

See also:

`lsim()`, `step()`, `impulse()`

**Examples**

```python
>>> yout, T = initial(sys, T, X0)
```

### 5.5.4 control.matlab.lsim

**control.matlab.lsim**(sys, **U=0.0, T=None, X0=0.0**)

Simulate the output of a linear system.

As a convenience for parameters **U**, **X0**: Numbers (scalars) are converted to constant arrays with the correct shape. The correct shape is inferred from arguments **sys** and **T**.

**Parameters**

- **sys** *(LTI (StateSpace, or TransferFunction))* – LTI system to simulate
- **U** *(array-like or number, optional)* – Input array giving input at each time **T** (default = 0).

If **U** is **None** or 0, a special algorithm is used. This special algorithm is faster than the general algorithm, which is used otherwise.

- **T** *(array-like)* – Time steps at which the input is defined, numbers must be (strictly monotonic) increasing.
- **X0** *(array-like or number, optional)* – Initial condition (default = 0).

**Returns**

- **yout** *(array)* – Response of the system.
- **T** *(array)* – Time values of the output.
- **xout** *(array)* – Time evolution of the state vector.

See also:

`step()`, `initial()`, `impulse()`
Examples

```python
>>> yout, T, xout = lsim(sys, U, T, X0)
```

5.6 Frequency-domain analysis

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<th>Description</th>
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<td><code>bode(syslist[, omega, dB, Hz, deg, ...])</code></td>
<td>Bode plot of the frequency response</td>
</tr>
<tr>
<td><code>nyquist(syslist[, omega, Plot, color, labelFreq])</code></td>
<td>Nyquist plot for a system</td>
</tr>
<tr>
<td><code>nichols(sys_list[, omega, grid])</code></td>
<td>Nichols plot for a system</td>
</tr>
<tr>
<td><code>margin(sysdata)</code></td>
<td>Calculate gain and phase margins and associated crossover frequencies</td>
</tr>
<tr>
<td><code>freqresp(sys, omega)</code></td>
<td>Frequency response of an LTI system at multiple angular frequencies.</td>
</tr>
<tr>
<td><code>evalfr(sys, x)</code></td>
<td>Evaluate the transfer function of an LTI system for a single complex number x.</td>
</tr>
</tbody>
</table>

5.6.1 control.matlab.bode

```python
control.matlab.bode (syslist[, omega, dB, Hz, deg, ...])
Bode plot of the frequency response
```

- **Parameters**
  - `sys` (LTI, or list of LTI) – System for which the Bode response is plotted and give. Optionally a list of systems can be entered, or several systems can be specified (i.e. several parameters). The sys arguments may also be interspersed with format strings. A frequency argument (array_like) may also be added, some examples: * >>> bode(sys, w) # one system, freq vector * >>> bode(sys1, sys2, ..., sysN) # several systems * >>> bode(sys1, sys2, ..., sysN, w) * >>> bode(sys1, 'plotstyle1', ..., sysN, 'plotstyleN') # + plot formats
  - `omega` (freq_range) – Range of frequencies in rad/s
  - `dB` (boolean) – If True, plot result in dB
  - `Hz` (boolean) – If True, plot frequency in Hz (omega must be provided in rad/sec)
  - `deg` (boolean) – If True, return phase in degrees (else radians)
  - `Plot` (boolean) – If True, plot magnitude and phase

Examples

```python
>>> sys = ss("1. -2; 3. -4", "5.; 7", "6. 8", "9.")
>>> mag, phase, omega = bode(sys)
```

Todo: Document these use cases

- ```python
  >>> bode(sys, w)
  ```
5.6.2 control.matlab.nyquist

control.matlab.nyquist(syslist, omega=None, Plot=True, color=None, labelFreq=0, *args, **kwargs)

Nyquist plot for a system
Plots a Nyquist plot for the system over a (optional) frequency range.

Parameters

- **syslist** *(list of LTI)* – List of linear input/output systems (single system is OK)
- **omega** *(freq_range)* – Range of frequencies (list or bounds) in rad/sec
- **Plot** *(boolean)* – If True, plot magnitude
- **color** *(string)* – Used to specify the color of the plot
- **labelFreq** *(int)* – Label every nth frequency on the plot
- ****kwargs** *(*args, **kwargs)* – Additional options to matplotlib (color, linestyle, etc)

Returns

- **real** *(array)* – real part of the frequency response array
- **imag** *(array)* – imaginary part of the frequency response array
- **freq** *(array)* – frequencies

Examples

```python
>>> sys = ss("1. -2; 3. -4", "5.; 7", "6. 8", "9.")
>>> real, imag, freq = nyquist_plot(sys)
```

5.6.3 control.matlab.nichols

control.matlab.nichols(sys_list, omega=None, grid=True)

Nichols plot for a system
Plots a Nichols plot for the system over a (optional) frequency range.

Parameters

- **sys_list** *(list of LTI, or LTI)* – List of linear input/output systems (single system is OK)
- **omega** *(array_like)* – Range of frequencies (list or bounds) in rad/sec
- **grid** *(boolean, optional)* – True if the plot should include a Nichols-chart grid. Default is True.
5.6.4 control.matlab.margin

control.matlab.margin(sysdata)

Calculate gain and phase margins and associated crossover frequencies

Parameters

sysdata (LTI system or (mag, phase, omega) sequence) –

- sys [StateSpace or TransferFunction] Linear SISO system
- mag, phase, omega [sequence of array_like] Input magnitude, phase (in deg.), and frequencies (rad/sec) from bode frequency response data

Returns

- gm (float) – Gain margin
- pm (float) – Phase margin (in degrees)
- wg (float) – Frequency for gain margin (at phase crossover, phase = -180 degrees)
- wp (float) – Frequency for phase margin (at gain crossover, gain = 1)

Margins are calculated for a SISO open-loop system.

If there is more than one gain crossover, the one at the smallest
margin (deviation from gain = 1), in absolute sense, is
returned. Likewise the smallest phase margin (in absolute sense)
is returned.

Examples

```python
>>> sys = tf(1, [1, 2, 1, 0])
>>> gm, pm, wg, wp = margin(sys)
```

5.6.5 control.matlab.freqresp

control.matlab.freqresp(sys, omega)

Frequency response of an LTI system at multiple angular frequencies.

Parameters

- sys (StateSpace or TransferFunction) – Linear system
- omega (array_like) – List of frequencies

Returns

- mag (ndarray)
- phase (ndarray)
- omega (list, tuple, or ndarray)

See also:
evalfr(), bode()
Notes

This function is a wrapper for StateSpace.freqresp and TransferFunction.freqresp. The output omega is a sorted version of the input omega.

Examples

```python
>>> sys = ss("1. -2; 3. -4", "5.; 7", "6. 8", "9.")
>>> mag, phase, omega = freqresp(sys, [0.1, 1., 10.])
>>> mag
array([[ 58.8576682 , 49.64876635, 13.40825927]])
>>> phase
array([[-0.05408304, -0.44563154, -0.66837155]])
```

Todo: Add example with MIMO system

```python
#>>> sys = rss(3, 2, 2) #>>> mag, phase, omega = freqresp(sys, [0.1, 1., 10.]) #>>> mag[0, 1, :] #array([55.43747231, 42.47766549, 1.97225895]) #>>> phase[1, 0, :] #array([-0.12611087, -1.14294316, 2.5764547]) #>>> # This is the magnitude of the frequency response from the 2nd input to the 1st output, and the phase (in radians) of the frequency response from the 1st input to the 2nd output, for s = 0.1i, i, 10i.
```

### 5.6.6 control.matlab.evalfr

control.matlab.evalfr(sys, x)

Evaluate the transfer function of an LTI system for a single complex number x.

To evaluate at a frequency, enter x = omega*j, where omega is the frequency in radians.

Parameters

- **sys** *(StateSpace or TransferFunction)*—Linear system
- **x** *(scalar)*—Complex number

Returns fresp

Return type ndarray

See also:

freqresp(), bode()

Notes

This function is a wrapper for StateSpace.evalfr and TransferFunction.evalfr.

Examples

```python
>>> sys = ss("1. -2; 3. -4", "5.; 7", "6. 8", "9.")
>>> evalfr(sys, 1j)
array([[ 44.8-21.4j]])
>>> # This is the transfer function matrix evaluated at s = i.
```
5.7 Compensator design

Todo: Add example with MIMO system

5.7.1 control.matlab.rlocus

control.matlab.rlocus(sys, kvect=None, xlim=None, ylim=None, plotstr='C0', Plot=True, PrintGain=True, grid=False, **kwargs)

Root locus plot

Calculate the root locus by finding the roots of 1+k*TF(s) where TF is self.num(s)/self.den(s) and each k is an element of kvect.

Parameters

- sys (LTI object) – Linear input/output systems (SISO only, for now).
- kvect (list or ndarray, optional) – List of gains to use in computing diagram.
- xlim (tuple or list, optional) – Set limits of x axis, normally with tuple (see matplotlib.axes).
- ylim (tuple or list, optional) – Set limits of y axis, normally with tuple (see matplotlib.axes).
- Plot (boolean, optional) – If True (default), plot root locus diagram.
- PrintGain (bool) – If True (default), report mouse clicks when close to the root locus branches, calculate gain, damping and print.
- grid (bool) – If True plot omega-damping grid. Default is False.

Returns

- rlist (ndarray) – Computed root locations, given as a 2D array
- klist (ndarray or list) – Gains used. Same as klist keyword argument if provided.

5.7.2 control.matlab.sisotool

control.matlab.sisotool(sys, kvect=None, xlim_rlocus=None, ylim_rlocus=None, plotstr_rlocus='C0', rlocus_grid=False, omega=None, dB=None, Hz=None, deg=None, omega_limits=None, omega_num=None, margins_bode=True, tvect=None)

Sisotool style collection of plots inspired by MATLAB’s sisotool. The left two plots contain the bode magnitude and phase diagrams. The top right plot is a clickable root locus plot, clicking on the root locus will change the gain of the system. The bottom left plot shows a closed loop time response.

Parameters
• `sys (LTI object)` – Linear input/output systems (SISO only)
• `kvect (list or ndarray, optional)` – List of gains to use for plotting root locus
• `xlim_rlocus (tuple or list, optional)` – control of x-axis range, normally
  with tuple (see matplotlib.axes)
• `ylim_rlocus (tuple or list, optional)` – control of y-axis range
• `plotstr_rlocus (Additional options to matplotlib)` – plotting style for
  the root locus plot(color, linestyle, etc)
• `rlocus_grid (boolean (default = False))` – If True plot s-plane grid.
• `omega (freq_range)` – Range of frequencies in rad/sec for the bode plot
• `dB (boolean)` – If True, plot result in dB for the bode plot
• `Hz (boolean)` – If True, plot frequency in Hz for the bode plot (omega must be
  provided in rad/sec)
• `deg (boolean)` – If True, plot phase in degrees for the bode plot (else radians)
• `omega_limits (tuple, list, .. of two values)` – Limits of the to generate
  frequency vector. If Hz=True the limits are in Hz otherwise in rad/s.
• `omega_num (int)` – number of samples
• `margins_bode (boolean)` – If True, plot gain and phase margin in the bode plot
• `tvect (list or ndarray, optional)` – List of timesteps to use for closed loop
  step response

Examples

```python
>>> sys = tf([1000], [1,25,100,0])
>>> sisotool(sys)
```

5.7.3 control.matlab.place

callbmatlab.place(A, B, p)
Place closed loop eigenvalues K = place(A, B, p)

Parameters

• `A (2-d array)` – Dynamics matrix
• `B (2-d array)` – Input matrix
• `p (1-d list)` – Desired eigenvalue locations

Returns

• `K (2-d array)` – Gain such that A - B K has eigenvalues given in p

Algorithm

This is a wrapper function for scipy.signal.place_poles, which
implements the Tits and Yang algorithm [1]. It will handle SISO,
MISO, and MIMO systems. If you want more control over the algorithm,
• use scipy.signal.place_poles directly.


• Limitations

  • The algorithm will not place poles at the same location more than $\text{rank}(B)$ times.

Examples

```python
>>> A = [[-1, -1], [0, 1]]
>>> B = [[0], [1]]
>>> K = place(A, B, [-2, -5])
```

See also:
place_varga(), acker()

5.7.4 control.matlab.lqr

control.matlab.lqr($A, B, Q, R[, N]$)

Linear quadratic regulator design

The lqr() function computes the optimal state feedback controller that minimizes the quadratic cost

$$J = \int_0^\infty (x'Qx + u'Ru + 2x'Nu)dt$$

The function can be called with either 3, 4, or 5 arguments:

• lqr(sys, Q, R)
• lqr(sys, Q, R, N)
• lqr($A, B, Q, R$)
• lqr($A, B, Q, R, N$)

where `sys` is an LTI object, and $A, B, Q, R$, and $N$ are 2d arrays or matrices of appropriate dimension.

Parameters

• $B$ ($A, \ldots$) – Dynamics and input matrices
• $\textbf{sys}$ (LTI (StateSpace or TransferFunction)) – Linear I/O system
• $R$ ($Q, \ldots$) – State and input weight matrices
• $N$ (2-d array, optional) – Cross weight matrix

Returns

• $K$ (2-d array) – State feedback gains
• $S$ (2-d array) – Solution to Riccati equation
• $E$ (1-d array) – Eigenvalues of the closed loop system
Examples

```python
>>> K, S, E = lqr(sys, Q, R, [N])
>>> K, S, E = lqr(A, B, Q, R, [N])
```

5.8 State-space (SS) models

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<td><code>rss</code> ([states, outputs, inputs])</td>
<td>Create a stable continuous random state space object.</td>
</tr>
<tr>
<td><code>drss</code> ([states, outputs, inputs])</td>
<td>Create a stable discrete random state space object.</td>
</tr>
<tr>
<td><code>ctrb</code> (A, B)</td>
<td>Controllability matrix</td>
</tr>
<tr>
<td><code>obsv</code> (A, C)</td>
<td>Observability matrix</td>
</tr>
<tr>
<td><code>gram</code> (sys, type)</td>
<td>Gramian (controllability or observability)</td>
</tr>
</tbody>
</table>

### 5.8.1 control.matlab ctrb

```python
control.matlab.ctrb(A, B)
```

Controllability matrix

- **Parameters**
  - B (A) – Dynamics and input matrix of the system
- **Returns**
  - C – Controllability matrix
- **Return type**
  - matrix

Examples

```python
>>> C = ctrb(A, B)
```

### 5.8.2 control.matlab obsv

```python
control.matlab.obsv(A, C)
```

Observability matrix

- **Parameters**
  - C (A) – Dynamics and output matrix of the system
- **Returns**
  - O – Observability matrix
- **Return type**
  - matrix

Examples

```python
>>> O = obsv(A, C)
```

### 5.8.3 control.matlab gram

```python
control.matlab.gram(sys, type)
```

Gramian (controllability or observability)

- **Parameters**
• **sys** (*StateSpace*) – State-space system to compute Gramian for

• **type** (*String*) – Type of desired computation. *type* is either ‘c’ (controllability) or ‘o’ (observability). To compute the Cholesky factors of gramians use ‘cf’ (controllability) or ‘of’ (observability)

**Returns** gram – Gramian of system

**Return type** array

**Raises**

• **ValueError** – * if system is not instance of StateSpace class * if type is not ‘c’, ‘o’, ‘cf’ or ‘of’ * if system is unstable (sys.A has eigenvalues not in left half plane)

• **ImportError** – if slycot routine sb03md cannot be found if slycot routine sb03od cannot be found

**Examples**

```python
>>> Wc = gram(sys, 'c')
>>> Wo = gram(sys, 'o')
>>> Rc = gram(sys, 'cf'), where Wc=Rc'\ast Rc
>>> Ro = gram(sys, 'of'), where Wo=Ro'\ast Ro
```

### 5.9 Model simplification

<table>
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<th>Function</th>
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<tr>
<td><code>minreal(sys[, tol, verbose])</code></td>
<td>Eliminates uncontrollable or unobservable states in state-space models or cancelling pole-zero pairs in transfer functions.</td>
</tr>
<tr>
<td><code>hsvd(sys)</code></td>
<td>Calculate the Hankel singular values.</td>
</tr>
<tr>
<td><code>balred(sys, orders[, method, alpha])</code></td>
<td>Balanced reduced order model of sys of a given order.</td>
</tr>
<tr>
<td><code>modred(sys, ELIM[, method])</code></td>
<td>Model reduction of sys by eliminating the states in ELIM using a given method.</td>
</tr>
<tr>
<td><code>era(YY, m, n, nin, nout, r)</code></td>
<td>Calculate an ERA model of order r based on the impulse-response data YY.</td>
</tr>
<tr>
<td><code>markov(Y, U, M)</code></td>
<td>Calculate the first M Markov parameters [D CB CAB …] from input U, output Y.</td>
</tr>
</tbody>
</table>

#### 5.9.1 control.matlab.minreal

`control.matlab.minreal(sys, tol=None, verbose=True)`

Eliminates uncontrollable or unobservable states in state-space models or cancelling pole-zero pairs in transfer functions. The output sysr has minimal order and the same response characteristics as the original model sys.

**Parameters**

• **sys** (*StateSpace or TransferFunction*) – Original system

• **tol** (*real*) – Tolerance

• **verbose** (*bool*) – Print results if True

**Returns** sysr – Cleaned model

**Return type** *StateSpace or TransferFunction*
5.9.2 control.matlab.hsvd

control.matlab.hsvd(sys)
Calculate the Hankel singular values.

Parameters  

- **sys** (*StateSpace*) – A state space system

Returns  

- **H** – A list of Hankel singular values

Return type  

- **Matrix**

See also:

- **gram()**

Notes

The Hankel singular values are the singular values of the Hankel operator. In practice, we compute the square root of the eigenvalues of the matrix formed by taking the product of the observability and controllability gramians. There are other (more efficient) methods based on solving the Lyapunov equation in a particular way (more details soon).

Examples

```python
>>> H = hsvd(sys)
```

5.9.3 control.matlab.balred

control.matlab.balred(sys, orders, method='truncate', alpha=None)
Balanced reduced order model of sys of a given order. States are eliminated based on Hankel singular value. If sys has unstable modes, they are removed, the balanced realization is done on the stable part, then reinserted in accordance with the reference below.


Parameters

- **sys** (*StateSpace*) – Original system to reduce
- **orders** (*integer or array of integer*) – Desired order of reduced order model (if a vector, returns a vector of systems)
- **method** (*string*) – Method of removing states, either 'truncate' or 'matchdc'.
- **alpha** (*float*) – Redefines the stability boundary for eigenvalues of the system matrix A. By default for continuous-time systems, alpha <= 0 defines the stability boundary for the real part of A's eigenvalues and for discrete-time systems, 0 <= alpha <= 1 defines the stability boundary for the modulus of A's eigenvalues. See SLICOT routines AB09MD and AB09ND for more information.

Returns  

- **rsys** – A reduced order model or a list of reduced order models if orders is a list

Return type  

- **StateSpace**

Raises

- **ValueError** – if **method** is not 'truncate' or 'matchdc'
• `ImportError` – if `slycot` routine `ab09ad`, `ab09md`, or `ab09nd` is not found
• `ValueError` – if there are more unstable modes than any value in `orders`

### Examples

```python
>>> rsys = balred(sys, orders, method='truncate')
```

#### 5.9.4 `control.matlab.modred`

`control.matlab.modred(sys, ELIM, method='matchdc')`

Model reduction of `sys` by eliminating the states in `ELIM` using a given method.

**Parameters**

- `sys` *(StateSpace)* – Original system to reduce
- `ELIM` *(array)* – Vector of states to eliminate
- `method` *(string)* – Method of removing states in `ELIM`: either 'truncate' or 'matchdc'.

**Returns** `rsys` – A reduced order model

**Return type** `StateSpace`

**Raises** `ValueError` – Raised under the following conditions:

- if `method` is not either 'matchdc' or 'truncate'
- if eigenvalues of `sys.A` are not all in left half plane (`sys` must be stable)

### Examples

```python
>>> rsys = modred(sys, ELIM, method='truncate')
```

#### 5.9.5 `control.matlab.era`

`control.matlab.era(YY, m, n, nin, nout, r)`

Calculate an ERA model of order `r` based on the impulse-response data `YY`.

**Note:** This function is not implemented yet.

**Parameters**

- `YY` *(array)* – `nout x nin` dimensional impulse-response data
- `m` *(integer)* – Number of rows in Hankel matrix
- `n` *(integer)* – Number of columns in Hankel matrix
- `nin` *(integer)* – Number of input variables
- `nout` *(integer)* – Number of output variables
- `r` *(integer)* – Order of model
Returns `sys` – A reduced order model `sys=ss(Ar,Br,Cr,Dr)`

Return type `StateSpace`

Examples

```python
>>> rsys = era(YY, m, n, nin, nout, r)
```

### 5.9.6 control.matlab.markov

`control.matlab.markov(Y, U, M)`

Calculate the first `M` Markov parameters `[D CB CAB ...]` from input `U`, output `Y`.

Parameters

- `Y` (*array_like*) – Output data
- `U` (*array_like*) – Input data
- `M` (*integer*) – Number of Markov parameters to output

Returns `H` – First `M` Markov parameters

Return type `matrix`

Notes

Currently only works for SISO

Examples

```python
>>> H = markov(Y, U, M)
```

### 5.10 Time delays

`pade(T[, n, numdeg])` Create a linear system that approximates a delay.

#### 5.10.1 control.matlab.pade

`control.matlab.pade(T, n=1, numdeg=None)`

Create a linear system that approximates a delay.

Return the numerator and denominator coefficients of the Pade approximation.

Parameters

- `T` (*number*) – time delay
- `n` (*positive integer*) – degree of denominator of approximation
- `numdeg` (*integer, or None (the default)) – If None, numerator degree equals denominator degree If >= 0, specifies degree of numerator If < 0, numerator degree is n+numdeg
Returns 

Returns `num, den` – Polynomial coefficients of the delay model, in descending powers of \(s\).

Return type array

Notes

Based on:

1. Algorithm 11.3.1 in Golub and van Loan, “Matrix Computation” 3rd. Ed. pp. 572-574

### 5.11 Matrix equation solvers and linear algebra

<table>
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<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>lyap(A, Q[, C, E])</code></td>
<td>X = lyap(A,Q) solves the continuous-time Lyapunov equation (AX + XA^T + Q = 0) where A and Q are square matrices of the same dimension. Further, Q must be symmetric.</td>
</tr>
<tr>
<td><code>dlyap(A, Q[, C, E])</code></td>
<td>dlyap(A,Q) solves the discrete-time Lyapunov equation (AXA^T - X + Q = 0) where A and Q are square matrices of the same dimension. Further Q must be symmetric.</td>
</tr>
<tr>
<td><code>care(A, B, Q[, R, S, E, stabilizing])</code></td>
<td>(X,L,G) = care(A,B,Q,R=None) solves the continuous-time algebraic Riccati equation</td>
</tr>
<tr>
<td><code>dare(A, B, Q, R[, S, E, stabilizing])</code></td>
<td>(X,L,G) = dare(A,B,Q,R) solves the discrete-time algebraic Riccati equation</td>
</tr>
</tbody>
</table>

### 5.11.1 control.matlab.lyap

```python
control.matlab.lyap (A, Q, C=None, E=None)
```

X = lyap(A,Q) solves the continuous-time Lyapunov equation

\[AX + XA^T + Q = 0\]

where A and Q are square matrices of the same dimension. Further, Q must be symmetric.

X = lyap(A,Q,C) solves the Sylvester equation

\[AX + XQ + C = 0\]

where A and Q are square matrices.

X = lyap(A,Q,None,E) solves the generalized continuous-time Lyapunov equation

\[AXE^T + EXA^T + Q = 0\]

where Q is a symmetric matrix and A, Q and E are square matrices of the same dimension.

### 5.11.2 control.matlab.dlyap

```python
control.matlab.dlyap (A, Q, C=None, E=None)
```

dlyap(A,Q) solves the discrete-time Lyapunov equation

\[AXA^T - X + Q = 0\]

where A and Q are square matrices of the same dimension. Further Q must be symmetric.

dlyap(A,Q,C) solves the Sylvester equation

\[AXQ^T - X + C = 0\]
where $A$ and $Q$ are square matrices.

dlyap($A, Q, None, E$) solves the generalized discrete-time Lyapunov equation

$$AXA^T - EXE^T + Q = 0$$

where $Q$ is a symmetric matrix and $A$, $Q$ and $E$ are square matrices of the same dimension.

### 5.11.3 control.matlab.care

```python
control.matlab.care(A, B, Q, R=None, S=None, E=None, stabilizing=True)
```

$(X, L, G) = care(A, B, Q, R)$ solves the continuous-time algebraic Riccati equation

$$A^T X + X A - X B R^{-1} B^T X + Q = 0$$

where $A$ and $Q$ are square matrices of the same dimension. Further, $Q$ and $R$ are symmetric matrices. If $R$ is None, it is set to the identity matrix. The function returns the solution $X$, the gain matrix $G = B^T X$ and the closed loop eigenvalues $L$, i.e., the eigenvalues of $A - B G$.

$(X, L, G) = care(A, B, Q, R, S, E)$ solves the generalized continuous-time algebraic Riccati equation

$$A^T X E + E^T X A - (E^T X B + S)R^{-1}(B^T X E + S^T) + Q = 0$$

where $A$, $Q$ and $E$ are square matrices of the same dimension. Further, $Q$ and $R$ are symmetric matrices. If $R$ is None, it is set to the identity matrix. The function returns the solution $X$, the gain matrix $G = R^{-1} (B^T X E + S^T)$ and the closed loop eigenvalues $L$, i.e., the eigenvalues of $A - B G$, $E$.

### 5.11.4 control.matlab.dare

```python
control.matlab.dare(A, B, Q, R, S=None, E=None, stabilizing=True)
```

$(X, L, G) = dare(A, B, Q, R)$ solves the discrete-time algebraic Riccati equation

$$A^T X A - X A^T B (B^T X B + R)^{-1} B^T X A + Q = 0$$

where $A$ and $Q$ are square matrices of the same dimension. Further, $Q$ is a symmetric matrix. The function returns the solution $X$, the gain matrix $G = (B^T X B + R)^{-1} B^T X A$ and the closed loop eigenvalues $L$, i.e., the eigenvalues of $A - B G$.

$(X, L, G) = dare(A, B, Q, R, S, E)$ solves the generalized discrete-time algebraic Riccati equation

$$A^T X A - E^T X E - (A^T X B + S)(B^T X B + R)^{-1}(B^T X A + S^T) + Q = 0$$

where $A$, $Q$ and $E$ are square matrices of the same dimension. Further, $Q$ and $R$ are symmetric matrices. The function returns the solution $X$, the gain matrix $G = (B^T X B + R)^{-1} (B^T X A + S^T)$ and the closed loop eigenvalues $L$, i.e., the eigenvalues of $A - B G$, $E$.

### 5.12 Additional functions

<table>
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<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>gangof4($P, C[, omega]$)</td>
<td>Plot the “Gang of 4” transfer functions for a system</td>
</tr>
<tr>
<td>unwrap($angle[, period]$)</td>
<td>Unwrap a phase angle to give a continuous curve</td>
</tr>
</tbody>
</table>

### 5.12.1 control.matlab.gangof4

```python
control.matlab.gangof4(P, C, omega=None)
```

Plot the “Gang of 4” transfer functions for a system
Generates a 2x2 plot showing the “Gang of 4” sensitivity functions [T, PS; CS, S]

**Parameters**
- $C(P)$ – Linear input/output systems (process and control)
- $\omega$ (array) – Range of frequencies (list or bounds) in rad/sec

**Returns**
- Return type: None

### 5.12.2 control.matlab.unwrap

`control.matlab.unwrap(angle, period=6.283185307179586)`

Unwrap a phase angle to give a continuous curve

**Parameters**
- `angle` (array_like) – Array of angles to be unwrapped
- `period` (float, optional) – Period (defaults to $2\pi$)

**Returns**
- `angle_out` – Output array, with jumps of period/2 eliminated
- Return type: array_like

**Examples**

```python
>>> import numpy as np
>>> theta = [5.74, 5.97, 6.19, 0.13, 0.35, 0.57]
>>> unwrap(theta, period=2 * np.pi)
[5.74, 5.97, 6.19, 6.413185307179586, 6.633185307179586, 6.8531853071795865]
```

### 5.13 Functions imported from other modules

- `linspace(start, stop[, num, endpoint, ...])` Return evenly spaced numbers over a specified interval.
- `logspace(start, stop[, num, endpoint, base, ...])` Return numbers spaced evenly on a log scale.
- `ss2zpk(A, B, C, D[, input])` State-space representation to zero-pole-gain representation.
- `tf2zpk(b, a)` Return zero, pole, gain (z, p, k) representation from a numerator, denominator representation of a linear filter.
- `zpk2ss(z, p, k)` Zero-pole-gain representation to state-space representation.
- `zpk2tf(z, p, k)` Return polynomial transfer function representation from zeros and poles

- genindex

### Development

You can check out the latest version of the source code with the command:
You can run a set of unit tests to make sure that everything is working correctly. After installation, run:

```
python setup.py test
```

Your contributions are welcome! Simply fork the GitHub repository and send a pull request.

**Links**

- [Issue tracker](https://github.com/python-control/python-control/issues)
- [Mailing list](http://sourceforge.net/p/python-control/mailman/)
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